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Solving Fuzzy Transportation Problems Using ASM Method and Zero Suffix Method

Aurora Nur Aini [a,1,*], Ali Shodiqin [a,2], Dewi Wulandari [a,3]

[a] Universitas PGRI Semarang, Jl. Sidodadi Timur No. 24, Semarang 50125, Indonesia
[1] auroranuraini@upgris.ac.id*; [2] alishodiqin@upgris.ac.id; [3] dewiwulandari@upgris.ac.id
* corresponding author

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ABSTRACT

The transportation problem is a special case for linear programming. Sometimes, the amount of demand and supply in transportation problems can change from time to time, and thus it is justified to classify the transportation problem as a fuzzy problem. This article seeks to solve the Fuzzy transportation problem by converting the fuzzy number into crisp number by ranking the fuzzy number. There are many applicable methods to solve linear transportation problems. This article discusses the method to solve transportation problems without requiring an initial feasible solution using the ASM method and the Zero Suffix method. The best solution for Fuzzy transportation problems with triangular sets using the ASM method was IDR 21,356,787.50, while the optimal solution using the Zero Suffix method was IDR 21,501,225.00.

1. Introduction (*Heading 1*)

The transportation problem is a branch of linear programming. The distances between consumers and producers require that the distribution process of goods be carried out to meet consumer needs. The company has to make the best decision to distribute goods from one place to another as effectively as possible and to spend as little as possible to get the maximum benefits.

Transportation models have been widely applied in logistics and supply chain for reducing cost. There have been many algorithms developed to solve the transportation problems when the exact cost coefficients and the supply and demand quantities are known. However, in fact, the cost coefficients and the supply - demand are fuzzy quantities in most cases. A fuzzy transportation problem is a transportation problem in which the transportation costs and supply and demand quantities are fuzzy quantities. [1]- [4]

In a transportation problem, if the requirement, availability and the cost per unit are represented by fuzzy numbers, the transportation problem is known as a Fully Fuzzy transportation problem or transportation problem with fuzzy environment. There are several methods that can be used to solve the fuzzy transportation problem, both in determining the initial feasible solution, namely the fuzzy MOMC (maximum supply with minimum cost) method, and the final direct solution, namely the fuzzy zero point method, the fuzzy zero suffix method, the fuzzy version of MODI method, fuzzy dual matrix approach, and etc. [2][5]

This paper will discuss the way to solve the fuzzy transportation problem where the cost, supply, and demand parameters are triangular fuzzy numbers by converting the fuzzy number into crisp number using ranking technique [1], and solve the linear transportation problem using ASM method and Zero Suffix method.

2. Method

Reference [5] of the fully fuzzy transformation problem is defined as follows:

$$\text{Minimize } Z \approx \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij} \quad (1)$$

Subject to,

$$\sum_{j=1}^{n} x_{ij} \approx a_i, i = 1, 2, ..., m$$

$$\sum_{i=1}^{m} x_{ij} \approx b_i, j = 1, 2, ..., n$$

$$\sum_{j=1}^{n} a_i \approx \sum_{i=1}^{m} b_i, i = 1, 2, ..., m, \quad j = 1, 2, ..., n$$

For all $x_{ij} \geq 0$

In this paper, we converted the fully fuzzy transportation problem into crisp problem using ranking technique by [1].

Definition: A fuzzy number $\tilde{A}$ is a triangular fuzzy number denoted by (δ, m, β) where δ, m and β are real number and its membership function $\mu_A(x)$ is given below.

$$\mu_A(x) = \begin{cases} 0, & \text{for } x \leq \delta \\ \dfrac{x-\delta}{m-\delta}, & \text{for } \delta \leq x \leq m \\ 1, & \text{for } x = m \\ \dfrac{\beta - x}{\beta - m}, & \text{for } m \leq x \leq \beta \\ 0, & \text{for } x \geq \beta \end{cases} \quad (2)$$

According to the definition of a triangular fuzzy number, let $\tilde{A} = \left(\underline{A}(r), \overline{A}(r)\right), (0 \leq r \leq 1)$ be a fuzzy number, the value of $M\left(\tilde{A}\right)$ is assigned to crisp number calculated as follows:

$$M_0^{Tri}(A) = \frac{1}{2}\int_0^1 \left\{\underline{A}(r) + \overline{A}(r)\right\} dr = \frac{1}{4}\left(2m + \delta + \beta\right) \quad (3)$$

Thus, we obtained the transportation problem on crisp number, which can be stated mathematically as follows:

$$\text{Minimize } Z = \sum_{i=1}^{m} X_{ij} \sum c_{ij} x_{ij} \quad (4)$$

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Subject to:

$$\sum_{j=1}^{n} x_{ij} \le a_i \qquad ;i = 1, 2, 3, ..., m$$

$$\sum_{i=1}^{m} x_{ij} \le b_j \qquad ;\text{j} = 1, 2, 3, ..., n$$

$$x_{ij} \ge 0$$

Where c_{ij} is the cost of transportation of a unit from the i-th source to the j-th destination. The quantity of x_{ij} is the quantity that can be distributed from the i-th origin to j-th destination.

3.1. ASM Method

ASM Method was proposed by Abdul Quddoos, Dr. Shakeel Javaid, and Prof. Mohd Masood Khalid [6]. ASM method can find the optimal solution directly without having to firstly find the initial basic feasible solution (IBFS).

The ASM methods are carried out in the following steps [6]:

Step 1: Constructing the transportation table from given transportation problem.

Step 2: Subtracting each row entries of the transportation table from the respective row minimum and then subtract each column entries of the resulting transportation table from respective column minimum.

Step 3: After having at least one zero in each row and in each column in the reduced cost matrix, the next step is selecting the first zero (row-wise) occurring in the cost matrix. Suppose (i, j)- th zero is selected, we need to count the total number of zeros (excluding the selected one) in the i-th row and j-th column. The subsequent step is selecting the next zero and counting the total number of zeros in the corresponding row and column in the same manner. The next step is continuing the same step for all zeros in the cost matrix.

Step 4: Choosing a zero for which the number of zeros counted in step 3 is minimum and supplying maximum possible amount to that cell. If tie occurs for some zeros in step 3, we need to choose a (k,l) -th zero breaking tie, such that the total sum of all the elements in the k th row and l th column is maximum. Afterwards, we need to allocate maximum possible amount to that cell.

Step 5: After performing step 4, we need to delete the row or column for further calculation where the supply from a given source is depleted or the demand for a given destination is satisfied.

Step 6: Then, we need to check whether the resultant matrix possesses at least one zero in each row and in each column. If not, we shall repeat step 2, or go to step 7 otherwise.

Step 7: Repeating step 3 to step 6 until and unless all the demands are satisfied and all the supplies are exhausted.

3.2. Zero Suffix Method

The zero suffix method proceeds as follows [7]- [8].

Step 1: Constructing the transportation table.

Step 2: Subtracting each row entries of the transportation table from the corresponding row minimum and subtracting each column entries of the transportation table from the corresponding column minimum.

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Step 3: When the reduced cost matrix contains at least one zero in each row and column, the next step is finding the suffix value of all the zeros in the reduced cost matrix by following simplification, and the suffix value is denoted by S,

Therefore S = {Adding the costs of nearest adjacent sides of zeros/ No. of costs added}

Step 4: Choosing the maximum of S. If it has one maximum value, we need to first supply to that demand corresponding to the cell. If it has more equal values, we need to select{ai, bj}and supply to that demand as maximum as possible.

Step 5: After conducting the above step, the exhausted demands (column) or supplies (row) shall be trimmed. The resultant matrix must possess at least one zero in each row and column, otherwise, we need to repeat step 2.

Step 6: Repeating step 2 to 4 until the optimal solution is obtained.

3. Results and Discussion

Numerical example was proceeded using the following fuzzy transportation problems:

Table 1. Fuzzy transportation problem

Source	**Destination**								**Supply**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
A1	1650; 1700; 1750	2000; 2200; 2100	1500; 1450; 1400	2100; 2000; 2150	1400; 1500; 1550	2000; 2150; 2100	2500; 2550; 2400	1650; 1600; 1700	**4917; 4925; 4933**
A2	3200; 3450; 3400	4600; 4400; 4500	1900; 1800; 2000	2700; 2600; 2750	3200; 3250; 3100	2300; 2400; 2500	3650; 3600; 3700	1300; 1350; 1400	**3591; 3599; 3607**
A3	3250; 3550; 3400	4500; 4400; 4550	1750; 1600; 1800	2000; 2100; 2300	2350; 2400; 2450	3000; 2850; 2900	4100; 4250; 4200	1450; 1550; 1500	**1311; 1319; 1327**
Demand	**346; 349; 352**	**392; 395; 398**	**247; 250; 253**	**66; 69; 72**	**7941; 7944; 7947**	**295; 298; 301**	**215; 218; 221**	**317; 320; 323**	**9819; 9843; 9867**

Using [1], we found the crisp number of Fuzzy supply, demand, and cost ($M^{Tri}(A)$):

Table 2. Finding $M^{Tri}(A)$

Cell	**Data 1**	**Data 2**	**Data 3**	**m**	δ	β	$M^{Tri}(A)$
A11	1650	1700	1750	1700	1650	1750	1700
A21	3200	3450	3400	3400	3200	3450	3362.5
A31	3250	3550	3400	3400	3250	3550	3400
A12	2000	2200	2100	2100	2000	2200	2100
A22	4600	4400	4500	4400	4500	4600	4475
A32	4500	4400	4550	4500	4400	4550	4487.5
A13	1500	1450	1400	1450	1400	1500	1450
A23	1900	1800	2000	1900	1800	2000	1900
A33	1750	1600	1800	1750	1600	1800	1725
A14	2100	2000	2150	2100	2000	2150	2087.5

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Cell	Data 1	Data 2	Data 3	m	δ	β	$M^{Tri}(A)$
A24	2700	2600	2750	2700	2600	2750	2687.5
A34	2000	2100	2300	2100	2000	2300	2125
A15	1400	1500	1550	1500	1400	1550	1487.5
A25	3200	3250	3100	3200	3100	3250	3187.5
A35	2350	2400	2450	2400	2350	2450	2400
A16	2000	2150	2100	2100	2000	2150	2087.5
A26	2300	2400	2500	2400	2300	2500	2400
A36	3000	2850	2900	2900	2850	3000	2912.5
A17	2500	2550	2400	2500	2400	2550	2487.5
A27	3650	3600	3700	3650	3600	3700	3650
A37	4100	4250	4200	4200	4100	4250	4187.5
A18	1650	1600	1700	1650	1600	1700	1650
A28	1300	1350	1400	1350	1300	1400	1350
A38	1450	1550	1500	1500	1450	1550	1500
D1	346	349	352	352	346	349	349
D2	392	395	398	398	392	395	395
D3	247	250	253	253	247	250	250
D4	66	69	72	72	66	69	69
D5	7941	7944	7947	7947	7941	7944	7944
D6	295	298	301	301	295	298	298
D7	215	218	221	221	215	218	218
D8	317	320	323	323	317	320	320
S1	4917	4925	4933	4933	4917	4925	4925
S2	3591	3599	3607	3607	3591	3599	3599
S3	1311	1319	1327	1327	1311	1319	1319
N (S+D)	9819	9843	9867	9867	9819	9843	9843

Table 3. Transportation table based on Table 1 and 2

Source	Destination								Supply
	1	2	3	4	5	6	7	8	
A1	1700	2100	1450	2087.5	1487.5	2087.5	2487.5	1650	4925
A2	3362.5	4475	1900	2687.5	3187.5	2400	3650	1350	3599
A3	3400	4487.5	1725	2125	2400	2912.5	4187.5	1500	1319
Demand	349	395	250	69	7944	298	218	320	**9843**

4.1. Finding the Optimal Solution of Fuzzy Transportation Problem using ASM Method

Using [6], the next step of finding the optimal solution is to subtract each row entries of the transportation Table 2 from the respective row minimum and subtract each column entries of the

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resulting transportation table from the respective column minimum. Then, we need to count the total number of zeros (excluding the selected one) in the i th row and j th column. The index in each zero at cell ij indicates the count of total zeros of i th row and j th column.

Table 4. Step 2 and 3 of ASM Method

Source	**Destination**								**Supply**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
A1	0 (0_5)	0 (0_5)	0 (0_5)	12.5	0 (0_5)	0 (0_5)	0 (0_5)	200	**4925**
A2	1762.5	2475	550	712.5	1800	412.5	1262.5	0 (0_1)	**3599**
A3	1650	2337.5	225	0 (0_1)	862.5	775	1650	0 (0_2)	**1319**
Demand	**349**	**395**	**250**	**69**	**7944**	**298**	**218**	**320**	**9843**

Since there are 2 zeros with the same index, we counted the total sum of all the elements in the k th row and l th column, and chose the maximum value between them.

A2 to 8: 1762,5+2475+550+712,5+1800+412,5+1262,5+200=9175

A3 to 4: 1650+2337,5+225+862,5+775+1650+12,5+712,5=8225

The next step is allocating 320 from A2 to 8. Since all of the demand from destination 8 was fulfilled by A2, column 8 was not used in the next step. Thus, we obtained the new transportation table as in the followings.

Table 5. New Transportation Table

Source	**Destination**							**Supply**
	1	**2**	**3**	**4**	**5**	**6**	**7**	
A1	0	0	0	12.5	0	0	0	**4925**
A2	1762.5	2475	550	712.5	1800	412.5	1262.5	**3599**
A3	1650	2337.5	225	0	862.5	775	1650	**1319**
Demand	**349**	**395**	**250**	**69**	**7944**	**298**	**218**	**9843**

We need to repeat the steps until all the demands are satisfied and all the supplies are exhausted. By performing these steps, we obtained the optimal allocation as follows:

Table 6. Optimal Transportation Table

Source	**Destination**								**Supply**
	1	2	3	4	5	6	7	8	
A1	1700	2100	1450	2087.5	1487.5	2087.5	2487.5	1650	
		395			4530				4925
A2	3362.5	4475	1900	2687.5	3187.5	2400	3650	1350	
	349				2414	298	218	320	3599
A3	3400	4487.5	1725	2125	2400	2912.5	4187.5	1500	
			250	69	1000				1319
Demand	349	395	250	69	7944	298	218	320	**9843**

Total Cost:

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$$= 395 \times 2100 + 4530 \times 1487,5 + 349 \times 3362,5 + 2414 \times 3187,5 + 298 \times 2400 +$$
$$218 \times 3650 + 320 \times 1350 + 250 \times 1725 + 69 \times 2125 + 1000 \times 2400$$
$$= 21.356.787,50$$

4.2. Finding the Optimal Solution of Fuzzy Transportation Problem using Zero Suffix Method

Based on [8] and [6], the first and second step of ASM Method and Zero suffix method are similar. Thus, we can use Table 3 for Zero Suffix method as well. According to [8], the 3rd step is about finding the suffix value.

Table 7. Step 3 of Zero Suffix Method

Source	Destination								Supply
	1	2	3	4	5	6	7	8	
A1	0(S=881.25)	0(S=825)	0(S=187.5)	12.5	0(S=604.17)	0(S=137.5)	0(S=487.5)	200	**4925**
A2	1762.5	2475	550	712.5	1800	412.5	1262.5	0(S=487.5)	**3599**
A3	1650	2337.5	225	0(S=600)	862.5	775	1650	0(S=825)	**1319**
Demand	**349**	**395**	**250**	**69**	**7944**	**298**	**218**	**320**	**9843**

Then, we need to allocate 349 from A1 to 1 since the cell A11 has the biggest Suffix value. We can, thus, eliminate Column 1 for the next step. Afterwards, we repeated Step 2 – 4 to obtain the optimal solution.

Table 8. Optimal Solution of Zero Suffix method

Source	Destination								Supply
	1	2	3	4	5	6	7	8	
A1	1700	2100	1450	2087.5	1487.5	2087.5	2487.5	1650	
	349	395			4181				4925
A2	3362.5	4475	1900	2687.5	3187.5	2400	3650	1350	
			250	69	2764	298	218		3599
A3	3400	4487.5	1725	2125	2400	2912.5	4187.5	1500	
					999			320	1319
Demand	349	395	250	69	7944	298	218	320	**9843**

Total Cost:

$$= 349 \times 1700 + 395 \times 2100 + 4181 \times 1487,5 + 250 \times 1900 + 69 \times 2687,5 +$$
$$2764 \times 3187,5 + 298 \times 2400 + 218 \times 3650 + 999 \times 2400 + 320 \times 1500$$
$$= 21.501.225$$

4. Conclusion

The costs incurred for distributing goods are not always the same every time. Likewise, the amount of demand and supply of goods often change over time. To solve those kinds of transportation problems, the use of fuzzy sets is of necessity. The optimal solution for Fuzzy transportation problems with triangular sets using the ASM method is IDR 21,356,787.50, while that using the Zero Suffix obtained the optimal solution of IDR 21,501,225.00. However, it is impossible to generalize that the ASM method is always better than the Zero Suffix method. For the purposes of decision making, it is necessary to calculate the number using several methods in order to obtain the most optimal results.

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