



Modeling and Forecasting Volatility in USD/GBP Exchange Rate

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ABSTRACT

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Rate changes can occur hourly, daily, or in large incremental shifts. These changes may impact firms by changing the cost of commodities imported from other countries and the demand for their goods among foreign consumers. Therefore, it is essential to forecast exchange rates to manage this business effect. This study aims to determine the best model for predicting volatility in the exchange rate between USD and GBP. In particular, we analyze exchange rates using the Autoregressive Integrated Moving Average (ARIMA) model and the volatility or variance model by Generalized Autoregressive Conditional Heteroscedasticity (GARCH). To determine the best model, the performance of each model is evaluated with several criteria, namely Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). The results show that EGARCH(1,1) has the best forecasting performance in the out-sample section because it can better capture out-sample data patterns with minimum RMSE, MAE, and MAPE.

1. Introduction

In finance, the exchange rate refers to the price at which one currency will be exchanged for another. The value of one country's currency with respect to another is also thought of as the exchange rate [1]. For instance, if the interbank exchange rate between the British Pound (GBP) and the US dollar (USD) is 0.78, it indicates that either £0.78 will be exchanged for \$1 or \$1 will be exchanged for £0.78. In this instance, it is said that the price of a dollar is £0.78 for a pound or that the price of a pound is \$1/0.78 for a dollar.

The economic activity, market interest rates, gross domestic product, and unemployment rate in each country frequently influence how much one currency will swap for another. They are established in the global financial market, where banks and other financial institutions trade currencies round-the-clock depending on these criteria and are known as market exchange rates. Rate changes may occur hourly, daily, or in big incremental shifts. This alteration may impact the company by altering the cost of commodities imported from another nation and the demand for their

goods among foreign consumers [2]. Hence, it is important to forecast the exchange rate for managing this business effect.

Exchange rate forecasting is challenging for academic researchers and business practitioners. Various models and time series approaches have been suggested to model and forecast the exchange rates. Unfortunately, empirical results often fail to meet theoretical expectations. Meese and Rogoff [3] show that the out-sample performance of many structural and time series models is no better than that of a simple random walk model. Their findings discourage many researchers in the area since the superiority of the random walk means unpredictability of the exchange rate. However, the models investigated in [3] are linear. Meanwhile, exchange rate data contain nonlinearities that linear models may not approximate well.

Exchange rate data usually include floating and volatility models. The floating component is always modeled using a mean model, such as the Autoregressive Integrated Moving Average (ARIMA) model class. The volatility is a nonlinearity form in variance that can be modeled using Autoregressive Conditional Heteroscedasticity (ARCH) or Generalized Autoregressive Conditional Heteroscedasticity (GARCH). Most volatility in business and economic data is influenced by stylized facts. The stylized facts that significantly impact modeling financial time series are fat tails, volatility clustering, and nonlinear dependence. It may influence the forecasting performance of the mean model [4]. Furthermore, GARCH is a time series model most applied in volatility [5].

This study aims to determine the best model for the exchange rates between USD and GBP forecasting. Specifically, we analyze the exchange rate using the mean model belonging to ARIMA and the volatility or variance model by GARCH. In order to determine the best model, the performance of each model is evaluated by some criteria, i.e., Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE).

2. Method

We now describe ARIMA as the mean model and GARCH as the volatility model for exchange rate forecasting.

2.1. Autoregressive Integrated Moving Average (ARIMA)

The ARIMA model is a method widely used for analyzing and forecasting time series data. This model was shown in published by Box and Jenkins in 1970. ARIMA is constructed by three parts, i.e., AR (autoregressive part), I (integrated part), and MA (moving average part). In order to estimate the ARIMA model, we have to follow four steps, including model identification, model estimation, diagnosis checking, and forecasting [6].

a. Model Identification

- As introduced above, ARIMA includes three parts. We denote the order of AR part as p , I part as d , MA part as q , hence we have ARIMA (p, d, q) . In order to apply the ARIMA model, we have to define p, d, q first.
- The stationary test of a time series can define the integrated part of the model. If the time series integrates at level 0, we have $I(d = 0)$. If the time series integrates at level 1, we have $I(d = 1)$. The popular method used for stationary tests is Dickey-Fuller.
- After the stationary test, we define p and q using the autocorrelation function (ACF) and partial autocorrelation function (PACF).
- AR model order p represents the observation at time t which is linearly related to previous time observations $t - 1, t - 2, \dots, t - p$. The equation form of the AR (p) or ARIMA $(p,0,0)$ model can be written as

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + e_t \quad (1)$$

where, $\phi_1, \phi_2, \dots, \phi_p$ is the autoregressive parameter, e_t is the error value at time t , and μ is a constant.

- The MA model describes an event where an observation at time t is expressed as a linear combination of a number of residuals. The equation for MA(q) or ARIMA (0,0, q) model can be written as

$$y_t = \mu + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \dots - \theta_q e_{t-q} \quad (2)$$

where $\theta_1, \theta_2, \dots, \theta_q$ is the moving average parameter, e_t is the error value at time t , and μ is a constant.

- ARMA model is a combination of AR and MA models that can be written in ARMA notation (p, q) or ARIMA ($p,0,q$). The equation of the ARMA model on the order p and q can be written as

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \dots - \theta_q e_{t-q} \quad (3)$$

- A common equation that represents non-stationary time series is the ARIMA (p,d,q) model, which can be written as

$$(1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p)(1 - B)^d y_t = (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q) e_t \quad (4)$$

where, ϕ_1, \dots, ϕ_p is the parameter of AR model; $(1 - B)^d$ is differencing level; y_t is the forecast value of period t ; $\theta_1, \dots, \theta_q$ is the parameter of MA model; and e_t is the residual value of period t .

b. Model Estimation

Estimating the parameters for Box–Jenkins models involves numerically approximating the solutions of nonlinear equations. For this reason, it is common to use statistical software designed to handle the approach – virtually all modern statistical packages feature this capability. The main approaches to fitting Box–Jenkins models are nonlinear least squares and maximum likelihood estimation. Maximum likelihood estimation is generally the preferred technique [7].

c. Diagnosis Checking

Model diagnostics for Box–Jenkins models is similar to model validation for nonlinear least squares fitting. The error term is assumed to follow the assumptions for a stationary univariate process. The residuals should be white noise (independent when their distributions are normal) drawings from a fixed distribution with a constant mean and variance. If the Box–Jenkins model is good for the data, the residuals should satisfy these assumptions. If these assumptions are not satisfied, we must fit a more appropriate model. That is, return to the model identification step and try to develop a better model. One way to assess if the residuals from the Box–Jenkins model follow the assumptions is to generate statistical graphics (including an autocorrelation plot) of the residuals. We could also look at the value of the Box-Ljung statistic [8]. This test is sometimes known as the Ljung–Box Q test, can defined as:

H_0 : The error term is independently distributed (the correlations in the population from which the sample is taken are 0, so that any observed correlations in the data result from the randomness of the sampling process / white noise).

H_1 : The error term is not independently distributed; it exhibits serial correlation (not white noise).

The test statistic is

$$Q = n(n + 2) \sum_{k=1}^h \frac{\hat{\rho}_k^2}{n-k}$$

where n is the sample size, $\hat{\rho}_k$ is the sample autocorrelation at lag k , and h is the number of lags being tested. Under H_0 , the statistic Q asymptotically follows a $\chi^2_{(h)}$. For significance level α , the critical region for rejection is

$$Q > \chi^2_{1-\alpha, h}$$

where $\chi^2_{1-\alpha, h}$ is the $(1 - \alpha)$ -quantile of the chi-squared distribution with h degrees of freedom [7].

d. Forecasting

After step 3, if the model is suitable, we continue forecasting by using the model chosen.

2.2. Generalized Autoregressive Conditional Heteroscedasticity (GARCH)

Since financial markets data often exhibit varying volatility, AR and MA models, that assume the conditional variances are constant, cannot capture the nonlinear dynamics. Linear models are unable to explain characteristics like volatility clustering, leverage effects, leptokurtosis and long memory in financial series [9]. Thus, we can employ a method to model nonlinear patterns as non-constant volatility. Autoregressive conditional heteroscedasticity (ARCH) and its derivative models are popularly utilized in modeling and forecasting asset dynamics. The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) (p, q) model allows for both autoregressive (AR) and moving average (MA) components in the heteroskedastic variance. This type of model aims to develop a volatility measure that can be used in financial decision-making. The GARCH model expresses the variance as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \tag{5}$$

where σ_t^2 is conditional variance, ε_t is return residual and $\alpha_0, \alpha_i, \beta_j$ are parameters to be estimated. The necessary condition for the positive variance is nonnegative value of $\alpha_0, \alpha_i, \beta_j$ parameters, and $\alpha_i + \beta_j$ is expected to be less than 1 for the model. In financial data series analysis, higher values of α_i coefficient implies a higher reaction of volatility to market shocks, while higher values of β_j coefficient shows the persistence of market shocks.

Brooks and Burke [10] recommend that the GARCH (1,1) model is sufficient to capture the volatility clustering in financial data. In this report, we use GARCH (1,1), with (6) for mean and (7) for variance.

$$r_t = \mu + \varepsilon_t \tag{6}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{7}$$

where r_t is the return at time t , μ is the average return and ε_t is the residual return. Since σ_t^2 is the variance at time t based on the past information at time $t - 1$, it is called conditional variance. The conditional variance in (7) is a function of three variables, i.e., a constant term (α_0), volatility news at the previous period (ε_{t-1}^2 or ARCH term), and the variance previous period (σ_{t-1}^2 or GARCH term). That means the conditional variance of ε at time t depends not only on the news about volatility from the previous period, but also on the last period conditional variance [11].

If the AR polynomial of the GARCH representation in (7) has a unit root, then we have an Integrated (IGARCH) model. Thus, IGARCH models are unit-root GARCH models. An IGARCH (1,1) variance equation can be written as

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + (1 - \alpha_1) \sigma_{t-1}^2 \tag{8}$$

The EGARCH or Exponential GARCH model was proposed by Nelson [12]. This model allows for asymmetric reaction of conditional variance to shocks. The specification conditional variance of EGARCH (1,1) is

$$\log \sigma_t^2 = \alpha_0 + \alpha_1 \left[\left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right] + \beta_1 \log \sigma_{t-1}^2 + \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \quad (9)$$

where γ denotes leverage effects, which accounts for the asymmetry of the model. If $\gamma < 0$ it means negative shocks (bad news) generate more volatility than positive shocks (good news), however, $\gamma > 0$ means positive news is more destabilizing than negative ones. $\gamma = 0$ shows the model to be symmetric. Since $\log \sigma_t^2$ may be negative, there are no sign restrictions for the parameters.

2.3. Model Evaluation

The performance of forecasting models is evaluated using four measures: RMSE, MAE, and MAPE, which are calculated using the following equations.

$$RMSE = \sqrt{\frac{\sum_{t=1}^T (y_t - \hat{y}_t)^2}{T}} \quad (10)$$

$$MAE = \sqrt{\frac{\sum_{t=1}^T |y_t - \hat{y}_t|}{T}} \quad (11)$$

$$MAPE = \frac{1}{T} \sqrt{\sum_{t=1}^T \left| \frac{y_t - \hat{y}_t}{y_t} \right|} \quad (12)$$

where T is the number of total observations, y_t and \hat{y}_t is the actual and forecast value at time t , respectively, $t \in T$. When comparing among models, the smallest RMSE, MAE, and MAPE are chosen as the best accurate forecast model.

3. Results and Discussion

We use a daily close exchange rate of USD to GBP over the period January 1, 2018 – December 1, 2022 or 1284 days. The data is obtained from the Yahoo finance website. For analysis, we divide the data into two parts, i.e., 1250 days for the in-sample part and 34 days for the out-sample part. We use in-sample part data to identify the model and estimate the model and out-sample part to evaluate the model.

3.1. Descriptive Data

First, we describe the in-sample part of the original data. There is 1 missing value among 1,250 of the total number of observations and the other descriptive statistics can be shown in Table 1.

Table 1. Descriptive Statistics of Daily Original Data

| Total Obs. | Number of Missing Value | Minimum | Maximum | Mean | Variance | Kurtosis | Skewness |
|------------|-------------------------|---------|---------|-------|----------|----------|----------|
| 1250 | 1 | 0.6972 | 0.9322 | 0.767 | 0.0015 | 1.0498 | 0.8050 |

Since there is one missing value in the original data, we investigate the location of the missing value. Hence, we can use plot series data to view the pattern of data and the location of missing values.

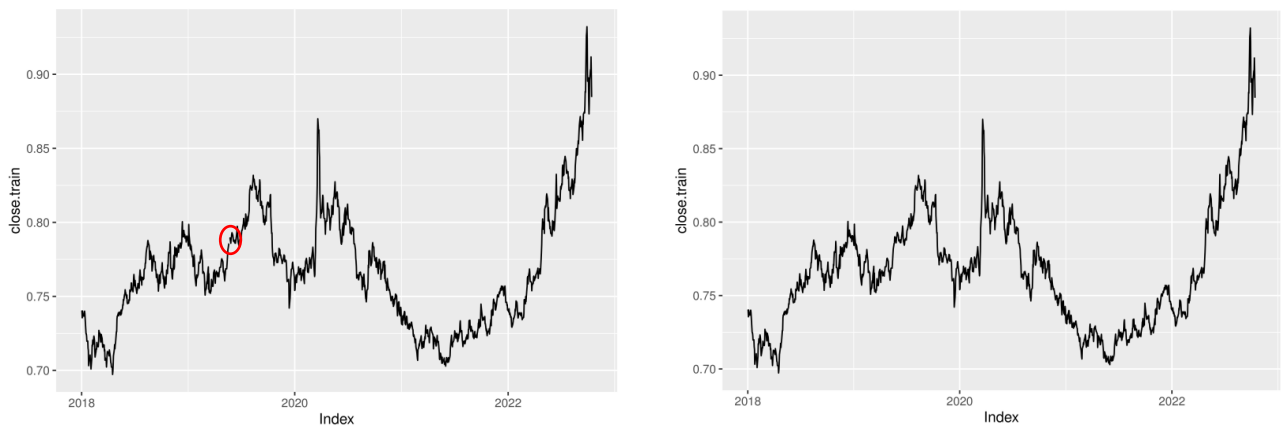


Fig. 1 Plot original in-sample data.

Fig. 1 is the plot of the original in-sample data. The left-hand panel shows the data series with the missing value signed by a red circle, by using the R code we got the missing is on May 22, 2019. Furthermore, we do imputation to missing value with the value around the previous and after the missing period. The right-hand panel shows the series plot without missing values. In addition, we can see that the data pattern is very fluctuating, there is a downward trend over the period of the mid-2020 and then it is going to an upward trend starting from mid-2021 up to the end of 2022. Besides that, some clusters of volatility can be shown in the plot. It indicates that the data is not a stationary process.

3.2. Fitting ARIMA Model

According to Fig. 1, we can see that the data is not a stationary process. In order to make it stationary, we can apply the first differencing to the data. The result of the stationary test is presented in Table 2 and the plot of the first differencing data is shown in Fig. 2.

Table 2. The Result of the Stationary Test

| Data | Dickey-Fuller | Lag order | P-Value |
|--------------------|---------------|-----------|---------|
| Original | -0.9352 | 10 | 0.9492 |
| First Differencing | -12.0240 | 10 | <0.01* |

*) Reject the null hypothesis

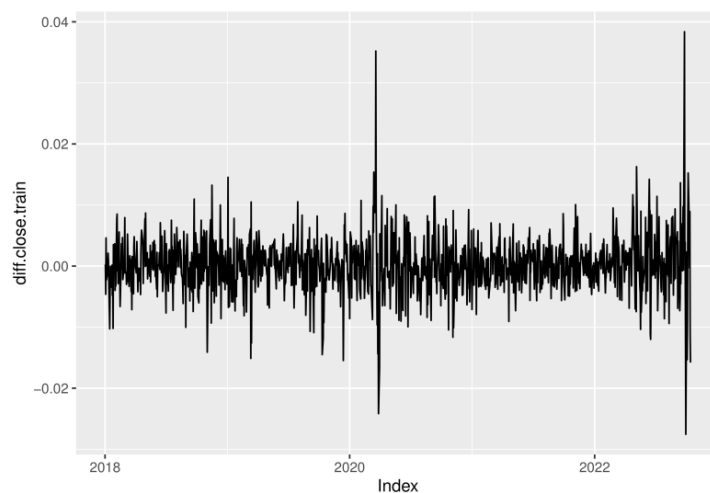


Fig. 2 Plot of first differencing data.

Table 2 presents the result of the Augmented Dickey-Fuller (ADF) test. The null hypothesis of this test is that there is a unit root in the data series (not a stationary process). At the same time, the alternative hypothesis is that the data series is a stationary process. The result shows that the original data is non-stationary because the p-value is larger than $\alpha = 0.05$. On the contrary, the first differencing data is a stationary process because the p-value is less than $\alpha = 0.05$. It is also shown in Fig. 2, that the first differencing data series look stationary and fluctuates around 0.

The next step is identifying the ARIMA model visually using ACF and PACF plots. The ACF and PACF plot is presented in Fig. 3 and Fig. 4.

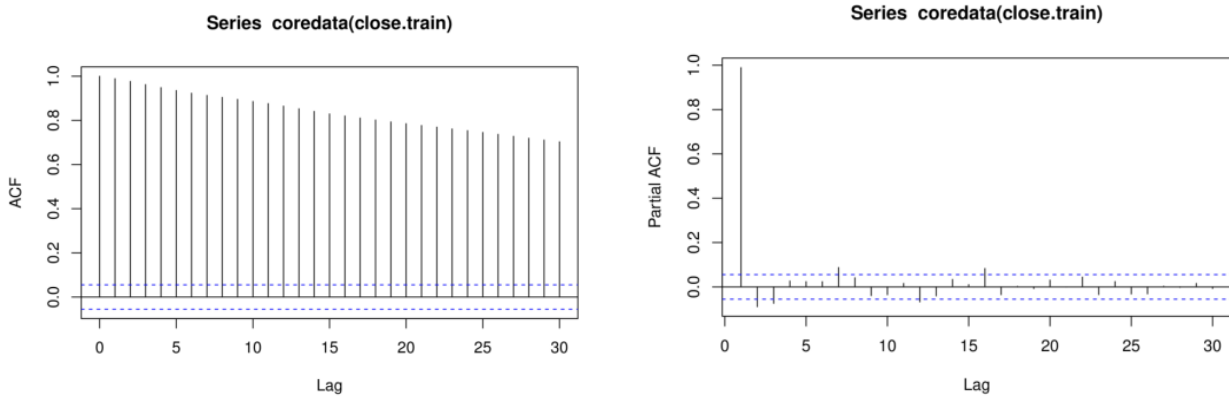


Fig. 3 ACF and PACF plot for original data.

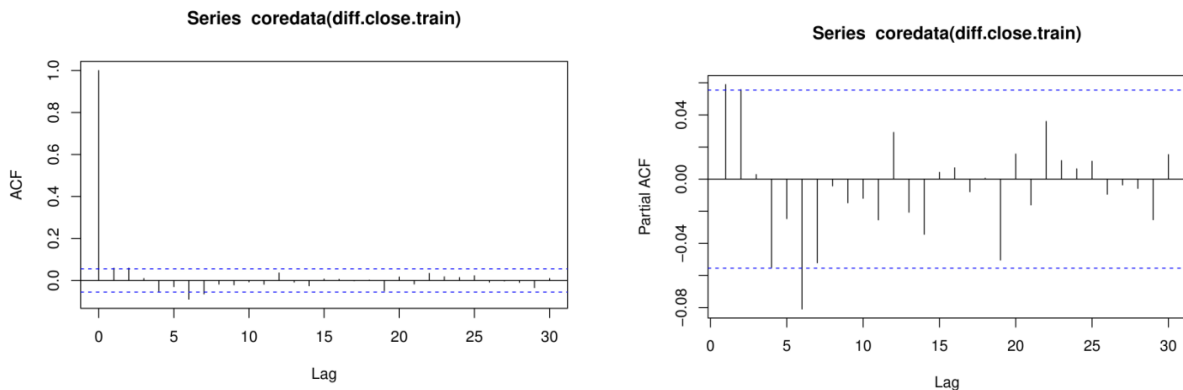


Fig. 4 ACF and PACF plot for first differencing data.

Fig. 3 shows ACF and PACF plots for the original data. In the left-hand panel, the ACF plot shows a prolonged decay. It indicates that the data might have influenced the current data long ago and shows that the data series is a non-stationary process. Since the data is not a stationary process, we consider ACF and PACF plots in Fig. 4 to identify a model for a non-stationary process. The order of the integrated (I) part is one because we only have the first difference. ACF plot in the left-hand panel of Fig. 4 almost looks like white noise, but there is a little bit cut off for lag one and lag 2. In the right-hand panel of Fig. 4, the PACF plot also almost looks like white noise, but we can also see a cut-off for lag 1 and 2. For this reason, we consider choosing ARIMA (2,1,2) as the candidate model. Besides that, we also apply the ‘auto. arima’ function in R code to consider another candidate model. The candidate model from ‘auto. arima’ function is ARIMA (0,1,2). Furthermore, the estimation parameter of model candidates is presented in Table 3. According to the Table 3, the candidate models can be written as

$$ARIMA (0,1,2) \quad (1 - B)y_t = (1 - 0.0554B - 0.0651B^2)e_t$$

$$ARIMA(2,1,2) \quad (1 - 1.6057B + 0.8545B^2)(1 - B)^d y_t = (1 + 1.5397B - 0.7912B^2)e_t$$

Table 3. Parameter Estimates of ARIMA Models

| Parameter Estimates | Model | |
|---------------------|-------------------------------|---------------|
| | auto.arima → ARIMA (0,1,2) | ARIMA (2,1,2) |
| AR 1 | - | 1.6057 |
| AR 2 | - | -0.8545 |
| MA 1 | 0.0554 | -1.5397 |
| MA 2 | 0.0651 | 0.7912 |

After we get the estimation of model candidates, the next step is diagnosis checking. The aim of diagnosis checking is to see whether the residuals follow the white noise process or meet the assumption of $IIDN \sim (0, \sigma^2)$. For this objective, we use the L-Jung Box test and the results are presented in Table 4.

Table 4. L-Jung Box (White Noise) Test Results

| Model | Q Test Statistic | Degrees of Freedom | P-Value |
|-------------------------------|------------------|--------------------|---------|
| auto.arima → ARIMA (0,1,2) | 0.000429 | 1 | 0.9835 |
| ARIMA (2,1,2) | 0.363430 | 1 | 0.5466 |

Table 4 shows the L-Jung Box Test for the residuals of model candidates. The null hypothesis is that the residuals are white noise. We can see that the p-value of both model candidates is larger than $\alpha=0.05$. It means both model candidates have the white noise residuals and are good models. Thus, we can consider these two models to forecast USD to GBP exchange rate.

However, Fig. 1 also informs us that there are multiple volatility clusters in the original data set. Because the ARIMA model is included in the linear model. It could be that ARIMA is unable to capture nonlinear patterns in the data. Hence, for the next part, we use GARCH as an alternative method to overcome volatility in the data series.

3.3. Fitting GARCH Model

As we mentioned in the previous part, data series have multiple volatilities. To estimate the volatility model, the mean return in equation (6) is first estimated to get the residuals. Then, squared residuals series and conditional variance are regressed on their lags and utilized to test the ARCH effect. The statistic descriptive of daily return data is presented in Table 5.

Table 5. Descriptive Statistics of Daily Return Data

| Total Obs. | Number of Missing Value | Minimum | Maximum | Mean | Variance | Kurtosis | Skewness |
|------------|-------------------------|---------|---------|--------|----------|----------|----------|
| 1249 | 0 | -0.0303 | 0.0423 | 0.0001 | 0.000003 | 5.7483 | 0.3462 |

According to Table 5, we can see that there is excess kurtosis in daily returns, which is 5.7483 larger than the normal value of 3. This can explain why heavier tails exist in the data and are distributed as leptokurtic. In other words, it may indicate that there is an ARCH effect. Thus, we can use the ARCH Lagrange-Multiplier (LM) test to check if there is an ARCH effect or not. The result of testing is reported in Table 6.

Table 6. ARCH-LM Test Results

| χ^2 | Degrees of Freedom | P-Value |
|----------|--------------------|---------|
| 12.615 | 1 | 0.00038 |

Table 6 shows the result of ARCH-LM test, the null hypothesis is that there is no ARCH effect. The result of this test is rejecting the null hypothesis because the p-value is less than $\alpha = 0.05$. It

means there is an ARCH effect so we can use the GARCH model. Furthermore, we estimate GARCH parameters and obtain the ARMA mean equation. Estimation results are reported in Table 7.

Table 7. Estimation Results of GARCH Models

| Parameter | GARCH(1,1) | | ARMA(1,1)-GARCH(1,1) | | ARMA(2,2)-GARCH(1,1) | |
|--------------------------|------------|--------|----------------------|--------|----------------------|--------|
| | Coef. | Prob. | Coef. | Prob. | Coef. | Prob. |
| <i>Mean Equation</i> | | | | | | |
| μ | 0.000076 | 0.600 | 0.00008 | 0.537 | 0.00007 | 0.627 |
| AR(1) | | | 0.89179 | 0.000 | -0.9998 | 0.000 |
| MA(1) | | | -0.9029 | 0.000 | 1.00404 | 0.000 |
| AR(2) | | | | | -0.9863 | 0.000 |
| MA(2) | | | | | 0.9998 | 0.000 |
| <i>Variance Equation</i> | | | | | | |
| α_0 | 0.000002 | 0.039 | 0.000002 | 0.021 | 0.000002 | 0.009 |
| α_1 | 0.097817 | 0.000 | 0.096922 | 0.000 | 0.097176 | 0.000 |
| β_1 | 0.845202 | 0.000 | 0.844473 | 0.000 | 0.842340 | 0.000 |
| γ | | | | | | |
| $\alpha_1 + \beta_1$ | 0.943019 | | 0.941395 | | 0.939516 | |
| L-Jung Box | 0.2659 | 0.6061 | 0.2524 | 0.6154 | 0.1563 | 0.6926 |
| Parameter | I-GARCH | | E-GARCH | | | |
| | Coef. | Prob. | Coef. | Prob. | | |
| <i>Mean Equation</i> | | | | | | |
| μ | 0.0001 | 0.451 | 0.0001 | 0.265 | | |
| <i>Variance Equation</i> | | | | | | |
| α_0 | 0.0000 | 0.658 | -0.365 | 0.000 | | |
| α_1 | 0.0862 | 0.000 | 0.035 | 0.007 | | |
| β_1 | 0.9138 | | 0.964 | 0.000 | | |
| γ | | | 0.046 | 0.000 | | |
| $\alpha_1 + \beta_1$ | 1 | | 0.999 | | | |
| L-Jung Box | 0.2797 | 0.5969 | 0.1132 | 0.7365 | | |

Table 7 shows estimation of some GARCH models i.e., GARCH(1,1), ARMA(1,1)-GARCH(1,1), ARMA(2,2)-GARCH(1,1), I-GARCH, and E-GARCH. Results show that the ARCH term (α_1), and GARCH term (β_1) are statistically significant in all of the models. This means that conditional variance correlates with lagged conditional variance and lagged square disturbance, or exchange rate news about volatility today has explanatory power on the next period's volatility. The sum of ARCH and GARCH terms in all models is close to 1. It indicates volatility shocks are quite persistent. L-Jung box test results show that all of the models have a white noise residual. Based on these parameter estimates, the models can be written as

GARCH (1,1)

Mean Equation

$$r_t = 0.000076 + \varepsilon_t$$

Variance Equation

$$\sigma_t^2 = 0.000002 + 0.097817\varepsilon_{t-1}^2 + 0.845202\sigma_{t-1}^2$$

ARMA(1,1)-GARCH(1,1)

Mean Equation

$$r_t = 0.00008 + 0.89179r_{t-1} - 0.9029\varepsilon_{t-1} + \varepsilon_t$$

Variance Equation

$$\sigma_t^2 = 0.000002 + 0.096922\varepsilon_{t-1}^2 + 0.844473\sigma_{t-1}^2$$

ARMA(2,2)-GARCH(1,1)

Mean Equation

$$r_t = 0.00007 - 0.9998r_{t-1} - 0.9863r_{t-2} + 1.00404\varepsilon_{t-1} + 0.9998\varepsilon_{t-2} + \varepsilon_t$$

Variance Equation

$$\sigma_t^2 = 0.000002 + 0.097176\varepsilon_{t-1}^2 + 0.842340\sigma_{t-1}^2$$

I-GARCH

Mean Equation

Variance Equation

$$r_t = 0.0001 + \varepsilon_t$$

$$\sigma_t^2 = 0.0862\varepsilon_{t-1}^2 + 0.9138\sigma_{t-1}^2$$

E-GARCH

Mean Equation

Variance Equation

$$r_t = 0.0001 + \varepsilon_t$$

$$\log \sigma_t^2 = -0.365 + 0.035 \left[\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right] + 0.964 \log \sigma_{t-1}^2$$

$$+ 0.046 \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$

3.4. Model Evaluation

In this part, we will evaluate the performance of models built in the previous part. We divide the evaluation into two parts, i.e., for in-sample data and out-sample data. Model evaluation for in-sample data based on RMSE, MAE, and MAPE criteria is reported in Table 8 and the performance models are visualized in Fig. 5.

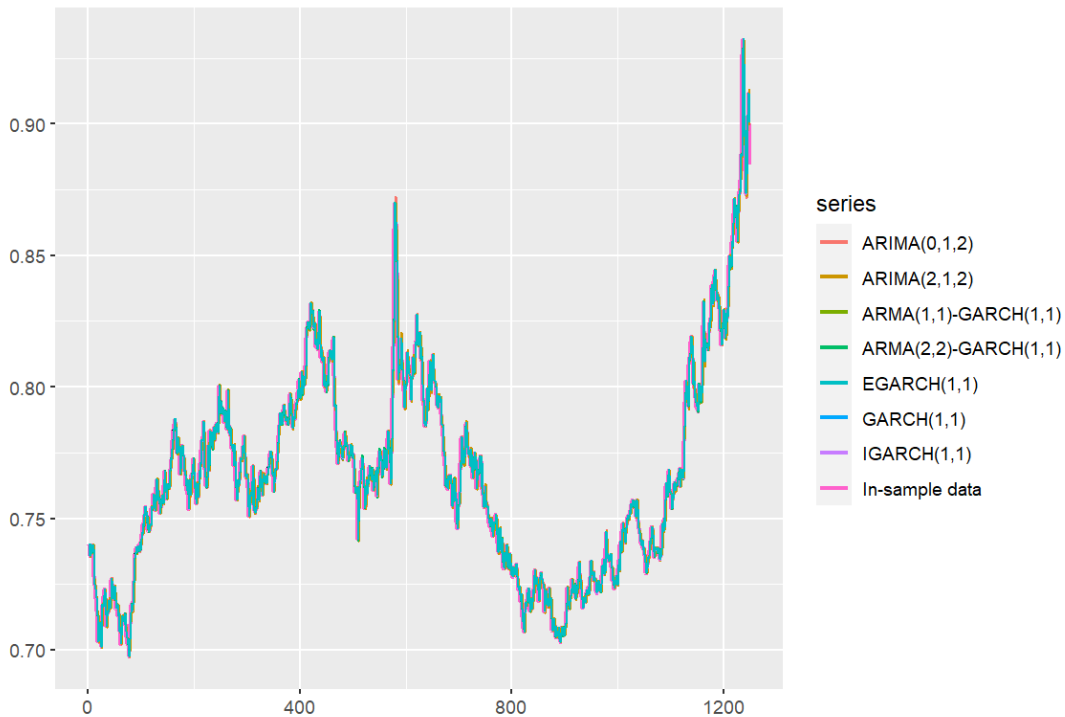


Fig. 5. Model performance for in-sample data.

According to Table 8, we can see that the criteria values of RMSE, MAE, and MAPE are very close among models and it's hard to decide which one is the best. It also shown in Fig. 5, all of the models are able to capture the fluctuation of in-sample data. However, in this case, we decide that the best model for in-sample data is ARIMA(2,1,2) with the smallest RMSE and MAPE. Since the final objective of the time series model is to forecast the future value, we also evaluate the performance of models to out-sample data. Model evaluation for out-sample data is reported in Table 9 and visualized in Fig. 6.

Table 8. Model Evaluation for In-Sample Data

| Model | RMSE | MAE | MAPE |
|----------------------|---------|---------|---------|
| ARIMA(0,1,2) | 0.00457 | 0.00325 | 0.00419 |
| ARIMA(2,1,2)* | 0.00455 | 0.00324 | 0.00417 |
| GARCH(1,1) | 0.00459 | 0.00324 | 0.00418 |
| ARMA(1,1)-GARCH(1,1) | 0.00458 | 0.00324 | 0.00418 |
| ARMA(2,2)-GARCH(1,1) | 0.00457 | 0.00324 | 0.00418 |
| IGARCH(1,1) | 0.00459 | 0.00324 | 0.00418 |
| EGARCH(1,1) | 0.00459 | 0.00324 | 0.00418 |

*) The smallest value (best model)

Table 9. Model Evaluation for Out-Sample Data

| Model | RMSE | MAE | MAPE |
|----------------------|--------|--------|--------|
| ARIMA(0,1,2) | 0.0301 | 0.0239 | 0.0282 |
| ARIMA(2,1,2) | 0.0289 | 0.0236 | 0.0279 |
| GARCH(1,1) | 0.0236 | 0.0204 | 0.0239 |
| ARMA(1,1)-GARCH(1,1) | 0.0237 | 0.0205 | 0.0240 |
| ARMA(2,2)-GARCH(1,1) | 0.0237 | 0.0205 | 0.0240 |
| IGARCH(1,1) | 0.0245 | 0.0205 | 0.0242 |
| EGARCH(1,1)* | 0.0236 | 0.0203 | 0.0238 |

*) The smallest value (best model)

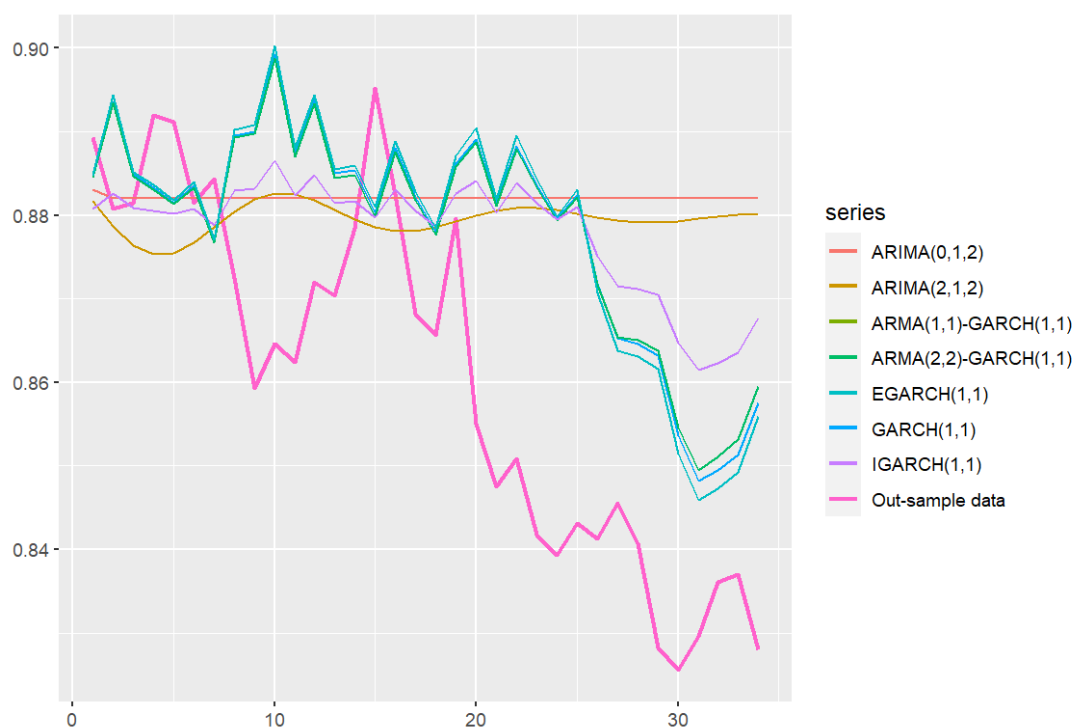


Fig. 6. Model performance for training data.

The out-sample data contains 34 days of the USD/GBP exchange rate i.e., along the period from 2022-10-17 up to 2022-12-01. Table 9 shows the model evaluation for out-sample data. Unlike in-sample data, the performance looks different among models for out-sample data. There are two models showing similar performance i.e., ARIMA(1,1)-GARCH(1,1) and ARIMA(2,2)-GARCH(1,1). These two models have identical values of RMSE, MAE, and MAPE. This is also visualized in Fig. 6 where we can see that these two models have a coinciding line. ARIMA(0,1,2) and ARIMA(2,1,2) cannot follow the out-sample data fluctuation and have larger criteria values. IGARCH (1,1) has a larger criteria value than the other kind of GARCH model, and its line is unable

to follow the fluctuation of out-sample data. GARCH(1,1) and EGARCH(1,1) have very close lines and criteria values. Their line is quite able to follow the fluctuation of out-sample data. However, in this case, EGARCH(1,1) has the smallest value of RMSE, MAE, and MAPE. Hence, we recommend that this model is the best to forecast USD/GBP exchange rate data.

4. Conclusion

In this report, the aim was to model and forecast USD/GBP exchange rate data using the mean model belonging to ARIMA and the variance or volatility model with kinds of GARCH. The results of this report show that in the out-sample part, EGARCH(1,1) has the best performance in forecasting because it is more able to capture the pattern of out-sample data with minimum RMSE, MAE, and MAPE. While, the mean model, ARIMA(0,1,2) and ARIMA(2,1,2) show bad performance for the out-sample part. Since this report aims to forecast, in this case, we recommend EGARCH(1,1) as the best model to forecast the USD/GBP exchange rate.

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