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Claim Reserving Estimation Using the Double Chain Ladder Method with the Bootstrap Approach

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ABSTRACT

The claim reserve is the amount of funds the insurance company must set aside to pay claims reported by policyholders. Estimation of claim reserves is carried out as a preventive step for failed payment if the reported claim exceeds the insurance company's capacity. The estimation of claim reserves in this study was performed using the double chain ladder method with a bootstrap approach. The data used was in the form of a run-off triangle of claim counts and claim amounts presented in incremental and cumulative form. The purpose of this research was to determine the estimated value of reported but not settled (RBNS) and incurred but not reported (IBNR) claim reserves through the bootstrap application on the double chain ladder method. After performing the double chain ladder calculation, the estimated RBNS claim reserves amounted to 6,828,456,000 and the IBNR amounted to 3,714,144,000. Meanwhile, using the bootstrap approach, the RBNS claim reserve estimate was 6,777,539,000 and the IBNR was 3,741,979,000. With the conclusion that the greater the nominal claim reserve allocated, the lower the chance of the company going bankrupt.

1. Introduction

The insurance industry is a business that provides insurance services in the form of risk management, risk reinsurance, insurance product offerings, consultations, and reinsurance. In short, insurance is the transfer of risk from one party to another in accordance with the agreed policy [1]. The establishment of an insurance company is based on the potential risks that may occur and result in future losses. It is an uncertain event that can cause positive or negative impacts. Negative uncertainty may lead to financial losses, prompting every individual to reduce negative risks and increase positive risks. A way to achieve this is by sharing the risks with the insurance company [2]. Nevertheless, the possibility of risk to arise is inevitable, be it positive or negative risk. In connection with that, insurance serves as a mechanism to transfer risk by providing financial guarantees to the insured party for unforeseen losses in the future. Based on performance records published by the Indonesian General Insurance Association (Asosiasi Asuransi Umum Indonesia, AAUI), 72 general insurance companies in Indonesia experienced positive growth in the first quarter of 2024, making a total premium income reaching IDR32 trillion. This value increased by approximately 25% compared to 2023 in the same period, amounting to IDR25 trillion. However, along with the

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increasing number of insurance customers, insurance companies face a higher risk of having to pay claims.

When experiencing risks, policyholders have the right to make demands for reimbursement and the company is obligated to reimburese them [3]. In the insurance field, this phenomenon is referred to as an insurance claim. Typically, insurance companies cannot immediately reimburse funds or pay the claims since they must gather information in the form of a first-hand review of the risks experienced, risk eligibility testing, estimation of claim costs, and more. The claim is stated as an outstanding claim during this period, meaning that the claim that has not been paid or settled. There are two types of outstanding claims: incurred but not reported (IBNR) and reported but not settled (RBNS). IBNR is a claim risk that has occurred but has not been reported to the insurance, while RBNS is a claim that has been reported but has not yet been paid [4]. The insurance company will compensate the policyholder if the claim requirements approve the claim. The nominal amounts of claims paid to policyholders are referred to as paid claims. Claims submitted by policyholders are the primary risk for insurance companies and liabilities that insurance companies must settle. The estimation process of calculating the total claim is called claim reserve estimation. The estimation of claim reserves is carried out as a preventive step for underpayment if the reported claim exceeds the insurance company's capabilities. Insurance companies are required to have an adequate claim reserve so that all reported claims may be settled [5], [6].

Chain ladder is one of the most familiar methods in estimating claim reserves due to its simplicity and the fact that it is distribution free, that it seems to work with almost no assumptions. The data used is the aggregate claim data in the run-off triangle, i.e., a set of claim payment data that occurs in a certain period of time (date of loss) and claim settlement time (date of payment) [7]. Although it is well-known and frequently used, the chain ladder method only delivers overall claim reserve estimation results. Thus, this research aimed to explain an extension algorithm that provides claim reserve estimation separately in the form of IBNR and RBNS. The method used in this scientific work drew on [8] and [9], forming a model by merging the run-off triangle of claim amounts and claim counts in estimating claim reserves. The model is later known as the double-chain ladder method. As the name implies, the chain ladder method was applied repeatedly twice, namely on the data of claim amounts and claim counts. Through a simple approach, the double chain ladder method was able to provide claim reserve estimation results separately in the form of RBNS and IBNR.

The bootstrap approach is a technique of resampling from a sample data set to generate a large number of bootstrap sample data. Each bootstrap sample yields a parameter that can describe the characteristics of the population [10]. It allows companies to deal with large and complex amounts of data regularly. The bootstrap approach was applied in this method to determine the predictive distribution value, which consists of the mean, prediction error, and percentile value of the claim reserve. Those predictive distribution values were then analyzed further to get an idea of the future development of claims. Although the amount of claim reserves that must be set up depends on the policies and capacity of the company itself, this information will assist the company in deciding the number of reserves, the extent of accuracy, and the ideal amount of claim reserves to be set up.

2. Theoretical Framework

2.1. Run-Off Triangle

A Run-off triangle is a data set of individual claims presented in aggregate form for a period of time such as monthly, quarterly, quarterly, and annually [11]. The data set can be either claims amount or claim counts presented in incremental and cumulative form. The year a risk occurs resulting in a claim report is referred to as the accident period and is denoted as i, given $1 \leq i \leq m$. The delay from the occurrence of a claim until the claim is reported or paid is called the period delay and is denoted as j, with $0 \leq j \leq m-1$. Suppose C_{ij} is a random variable that defines the number of claims incurred in accident period i and paid in period delay j, given $1 \leq i \leq m$ dan $0 \leq j \leq m-1$. Table 1 illustrates the incremental run-off triangle in the form of an $n \times n$ matrix that presents paid claims data. The number of claims in observation $i + j > m$ in the green cells are referred to as the future triangle.

Table 1. Run-off Triangle and Future Triangle Data in Incremental Form

Accident Period (i)	Period Delay (j)					
	0	***1***	**…**	***j***	**…**	***m-1***
1	C_{10}	C_{11}	…	C_{1j}	…	$C_{1,m-1}$
2	C_{20}	C_{21}	…	C_{2j}	…	$C_{2,m-1}$
…	…	…	…	…	…	…
i	C_{i0}	C_{i1}	…	C_{ij}	…	$C_{i,m-1}$
…	…	…	…	…	…	…
m	C_{m0}	C_{m1}	…	C_{mj}	…	$C_{m,m-1}$

Suppose D_{ij} is a random variable that defines the amount of cumulative claims incurred in accident period i and paid in period delay j, given $1 \le i \le m, 0 \le j \le m-1$, and $i + j \le m$. The value of cumulative claims will be calculated up to the period delay $m - 1$. The cumulative run-off triangle D_{ij} is formed based on the incremental run-off triangle C_{ij} using the (1).

$$D_{ij} = \sum_{l=0}^{j} C_{il}. \tag{1}$$

The incremental run-off triangle of claim counts is presented in Table 2.

Table 2. Run-off Triangle and Future Triangle Data in Cumulative Form

Accident Period (i)	Period Delay (j)					
	0	***1***	**…**	***j***	**…**	***m-1***
1	D_{10}	D_{11}	…	D_{1j}	…	$D_{1,m-1}$
2	D_{20}	D_{21}	…	D_{2j}	…	$D_{2,m-1}$
…	…	…	…	…	…	…
i	D_{i0}	D_{i1}	…	D_{ij}	…	$D_{i,m-1}$
…	…	…	…	…	…	…
m	D_{m0}	D_{m1}	…	D_{mj}	…	$D_{m,m-1}$

2.2. Chain Ladder Method

The chain ladder method is one of the most well-known and simple methods in estimating claim reserves, as the data used does not have to meet a certain distribution assumption [12]. Here, there is a loss development factor $\lambda_1, \dots\dots, \lambda_{m-1} > 0$ given $1 \le i \le m$ and $1 \le j \le m - 1$. The estimation value of loss development factor can be obtained by solving (2),

$$\hat{\lambda}_j = \frac{\sum_{i=1}^{m-j} D_{i,j}}{\sum_{i=1}^{m-j} D_{i,j-1}}. \tag{2}$$

The total cumulative claims $\widehat{D}_{i,j}$ for the future claims for $i + j > m$, is calculated using (3):

$$\widehat{D}_{i,j} = D_{i,j-1} \times \hat{\lambda}_j. \tag{3}$$

The estimation of claim reserves per accident period i, given $1 < i \le m$ and $1 \le j < m - 1$, is obtained using (4).

$$\hat{R}_i = \widehat{D}_{i,m-1} - \widehat{D}_{i,m-i}. \tag{4}$$

The final step in estimating the value of the outstanding claim reserves $\hat{R}$ with the chain ladder method is using (5):

$$\hat{R} = \sum_{i=2}^{m} \hat{R}_i. \tag{5}$$

2.3. Double Chain Ladder Method

As the name implies, the chain ladder method in the run-off triangle will be carried out twice, namely on the claim counts and claim amounts [13]. The data used were aggregated incurred counts and aggregated claim payments. In the aggregated incurred counts, $N_{ij}, (i,j) \in$ I, given N_{ij} is the number of claims that occurred in accident period i and were reported in the year of period delay $i + j$, as $\mathrm{I} = \{(i,j) : i = 1, \dots, m;\ j = 0, \dots, m-1; i + j \leq m\}$. In the claim counts triangle, period delay represents the reporting delay since the claim occurred. Meanwhile, in the aggregated claim payments, $X_{ij}, (i,j) \in$ I, given X_{ij}, is the total claim payment incurred in accident period i and paid in period delay j since the claim was incurred. In the claim amounts triangle, period delay represents the delay in reporting and payment since the claim occurred. The index of the run-off triangle illustrated is shown in Fig. 1.

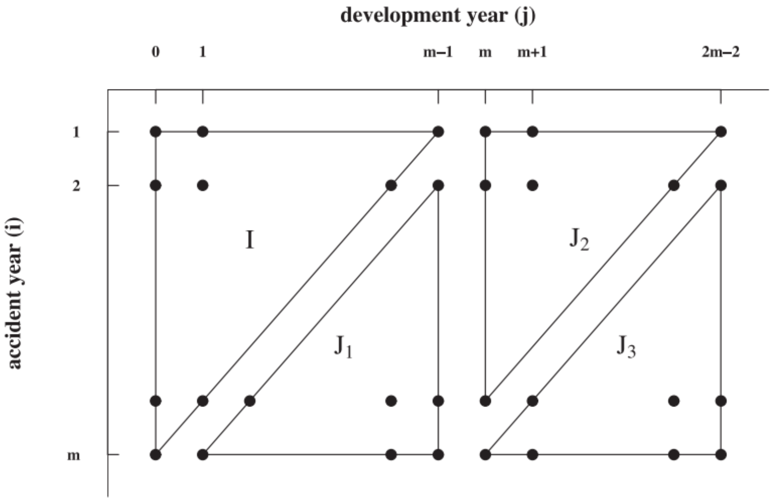

Fig. 1 Run-off triangle index set of aggregate claims data.

$\mathrm{I} = \{(i,j) : i = 1, \dots, m;\ j = 0, \dots, m-1; so\ i + j \leq m\}$

$\mathrm{J}_1 = \{(i,j) : i = 2, \dots, m;\ j = 0, \dots, m-1; so\ i + j = m+1, \dots, m+d\}$

$\mathrm{J}_2 = \{(i,j) : i = 1, \dots, m;\ j = m, \dots, m+d; so\ i + j = m+1, \dots, m+d\}$

$\mathrm{J}_3 = \{(i,j) : i = 2, \dots, m;\ j = m, \dots, m+d; so\ i + j = m+d+1, \dots, 2m+d\}$

The estimation of claim reserves using the chain ladder method will generate predicted values along the index triangle J_1. However, estimation using the double chain ladder method will generate values along the index $\mathrm{J}_1 \cup \mathrm{J}_2 \cup \mathrm{J}_3$ automatically. The estimated claim reserves that will fulfill index $\mathrm{J}_2 \cup \mathrm{J}_3$ is referred to as the tail [14].

2.4. Double Chain Ladder Parameter Estimation

Parameter estimation in the double chain ladder method is determined based on the run-off triangle of claim counts and claim amounts. Thus, there are two types of parameters derived: parameters for the run-off triangle of claim counts $(\hat{\alpha}_i, \hat{\beta}_j)$ and claims amounts $(\tilde{\alpha}_i, \tilde{\beta}_j)$. The value of α_i denotes the proportion of the ultimate number of claims in accident period *I*, while β_j denotes the proportion of ultimate number of claims in period delay *j*. The chain ladder algorithm produces estimates of development factors, $\hat{\lambda}_j, j = 1, \dots, m-1$, which the estimation of parameter $\hat{\beta}_j$ for $j = 0, \dots, m-1$ for run-off triangle of claim counts is obtained using (6) and (7):

$$\hat{\beta}_0 = \frac{1}{\prod_{l=1}^{m-1} \hat{\lambda}_l}, \tag{6}$$

$$\hat{\beta}_j = \frac{\hat{\lambda}_{j-1}}{\prod_{l=j}^{m-1} \hat{\lambda}_l}, \tag{7}$$

given $j = 1, \dots, m-1$. The estimation of parameter α_i is obtained using (8):

$$\hat{\alpha}_i = \sum_{j=0}^{m-i} N_{ij} \prod_{j=m-i+1}^{m-1} \hat{\lambda}_j, \tag{8}$$

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The same approach can be applied in determining the parameters for run-off triangle of claim amounts $(\tilde{\alpha}_i, \tilde{\beta}_j)$,

$$\tilde{\beta}_0 = \frac{1}{\prod_{l=1}^{m-1} \tilde{\lambda}_l}, \tag{9}$$

$$\tilde{\beta}_j = \frac{\tilde{\lambda}_{j-1}}{\prod_{l=j}^{m-1} \tilde{\lambda}_l}, \tag{10}$$

$$\tilde{\alpha}_i = \sum_{j=0}^{m-i} X_{ij} \prod_{j=m-i+1}^{m-1} \tilde{\lambda}_j. \tag{11}$$

2.5. RBNS Delay Parameter Estimation

The estimated value of parameter β_j, obtained from the run-off triangle of claim counts and claim amounts, is then used to determine the parameter $\pi = \{\pi_k; k = 0, \dots, m-1\}$ through the system of linear equations as in (12):

$$\begin{pmatrix} \tilde{\beta}_0 \\ \vdots \\ \vdots \\ \tilde{\beta}_{m-1} \end{pmatrix} = \begin{pmatrix} \hat{\beta}_0 & 0 & \cdots & 0 \\ \hat{\beta}_1 & \hat{\beta}_0 & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ \hat{\beta}_{m-1} & \cdots & \hat{\beta}_1 & \hat{\beta}_0 \end{pmatrix} \begin{pmatrix} \pi_0 \\ \vdots \\ \vdots \\ \pi_{m-1} \end{pmatrix}, \tag{12}$$

Given, $\hat{\pi}$ is the solution of the given linear equation with $\hat{\pi} = \{\hat{\pi}_k; k = 0, \dots, m-1\}$. The value of $\hat{\pi}_k$ may be negative and the total value may be greater than one. The value of $\hat{\pi}$ is adjusted as $\hat{p}_k$, so that it meets the conditions $\sum_{l=0}^{m-1} p_l = 1$ and $0 \leq p_l \leq 1$. The parameter $\hat{p}_k$ is estimated for $k = 0, \dots, d$. The estimation of the maximum delay parameter, d, is derived by counting the number of consecutive values of $\hat{\pi}_k \geq 0$. The formula for estimating the delay parameter for $k = 0, \dots, d-1$ is shown in (13):

$$\hat{p}_k = \hat{\pi}_k;\ k = 0, \dots, d-1 \tag{13}$$

Therefore, the value of $\hat{p}_k$ for $k = d$ is estimated using (14);

$$\hat{p}_d = 1 - \sum_{k=0}^{d-1} \hat{p}_k. \tag{14}$$

given $\hat{p} = (\hat{p}_0, \dots, \hat{p}_{d-1}, 1 - \sum_{l=0}^{d-1} \hat{p}_k)$ is the delay probabilities. The remaining $\hat{p}_k$ are set as 0, so that $\hat{p}_{d+1} = \cdots = \hat{p}_{m-1} = 0$ and the condition $\sum_{l=0}^{m-1} p_k = 1$ is satisfied.

2.6. Individual Payment Parameter Estimation

In this section, the mean factor and the parameter that measures the inflation rate in accident period i will be estimated. The estimation value of the mean factor $\hat{\mu}$ is determined using (15):

$$\hat{\mu} = \frac{\tilde{\alpha}_1}{\hat{\alpha}_1}, \tag{15}$$

Given, γ is a parameter that measures the inflation rate with $\gamma = \{\gamma_i; i = 1, \dots, m\}$. The estimation value of γ is obtained by using (16):

$$\hat{\gamma}_i = \frac{\tilde{\alpha}_i}{\hat{\alpha}_i \hat{\mu}};\ i = 1, \dots, m. \tag{16}$$

The number of parameters that will be generated may lead to the model being over-parametrized. Therefore, a simple solution to ensure that the parameters can be identified easily is to set the value as $\gamma_1 = 1$.

2.7. Claim Reserve Estimation Using Double Chain Ladder Method

The estimation of claim reserves using the double chain ladder method was carried out separately on RBNS and IBNR. For the RBNS, it is assumed that the amount of claims payments is closely related to the number of claims reported by policyholders. Therefore, the estimation of RBNS claim reserves requires data on the number of original claims N_{ij} in the upper triangle of index I. The point forecast for the RBNS claims reserve component, $\hat{X}_{ij}^{RBNS}$ is determined using (17):

$$\hat{X}_{ij}^{RBNS} = \sum_{k=i-m+j}^{j} N_{i,j-k} \hat{p}_k \hat{\mu} \hat{\gamma}_i. \tag{17}$$

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The point forecast of the RBNS component is vary along the index $J_1 \cup J_2$. The estimation of RBNS claim reserves based on the future year is determined by summing the point forecasts along the diagonal of the index $J_1 \cup J_2$. In contrast to RBNS, the estimation of IBNR claim reserves requires data on claim counts in the future triangle $\hat{N}_{i,j}$. First, determine the estimated number of claims that will be reported in the future, $\hat{N}_{i,j}$, on the index triangle $(i,j) \in J_1$ using the chain ladder method in (3). Then, determine the point forecast for the IBNR component, $\hat{X}_{ij}^{IBNR}$, using (18).

$$\hat{X}_{ij}^{IBNR} = \sum_{l=0}^{i-m+j-1} \hat{N}_{i,j-k} \hat{p}_k \hat{\mu} \hat{\gamma}_i. \quad (18)$$

Automatically, the point forecast of the IBNR claim reserve component will vary along the index $J_1 \cup J_2 \cup J_3$. The estimation of IBNR claim reserves based on the future year is determined by summing up the point predictions along the diagonal at index $J_1 \cup J_2 \cup J_3$. The estimation of claim reserves using the double chain ladder method, $\hat{X}_{ij}^{DCL}$, is the total RBNS and IBNR claim reserves expressed in (19):

$$\hat{X}_{ij}^{DCL} = \hat{X}_{ij}^{RBNS} + \hat{X}_{ij}^{IBNR}. \quad (19)$$

2.8. Bootstrap Approach on Double Chain Ladder Method

The bootstrap method is a technique of resampling from a set of sample data in turn so that a large amount of bootstrap sample data is produced [15]. The objectives of the bootstrap approach application in this final project were to determine the predictive distribution, namely the mean, prediction error, and percentile value of the claim reserves that can illustrate the distribution of the claim data.

2.8.1. Bootstrap Algorithm for RBNS

The estimation of RBNS claim reserves in the bootstrap approach will also satisfy the values along index $J_1 \cup J_2$. Fig. 2 illustrates the flowchart of the steps of the bootstrap approach for RBNS claims.

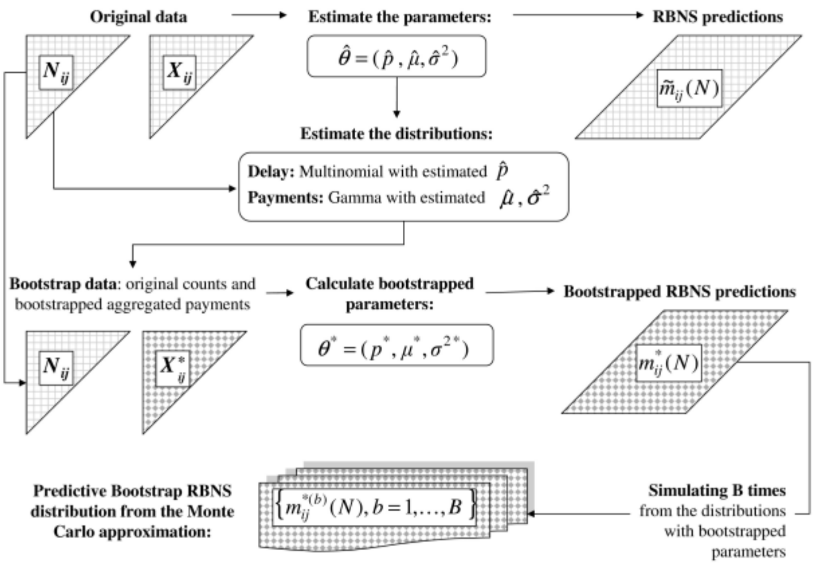

Fig. 2 Bootstrap Algorithm for RBNS

The Fig. 2 shows that the original claim data (N_{ij}, X_{ij}) is used to estimate the parameter $\hat{\theta}$ so that the RBNS claim reserve along index $J_1 \cup J_2$ is derived. Furthermore, using the estimated parameter, the bootstrap can be applied to the original claim amounts X_{ij}. Thus, the bootstrap claim amounts X_{ij}^* are derived. The run-off triangle (N_{ij}, X_{ij}^*) is then used to estimate the bootstrap parameter θ^* using the double chain ladder method so that the bootstrap RBNS claim reserves are derived. To derive the RBNS predictive distribution value, repeat the steps B times, where B is the number of bootstrap simulations.

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2.8.2. Bootstrap Algorithm for IBNR

The estimation of IBNR claim reserves in the bootstrap approach will satisfy the values along index $J_1 \cup J_2 \cup J_3$. The Fig. 3 shows the steps of estimating an IBNR claim in detail.

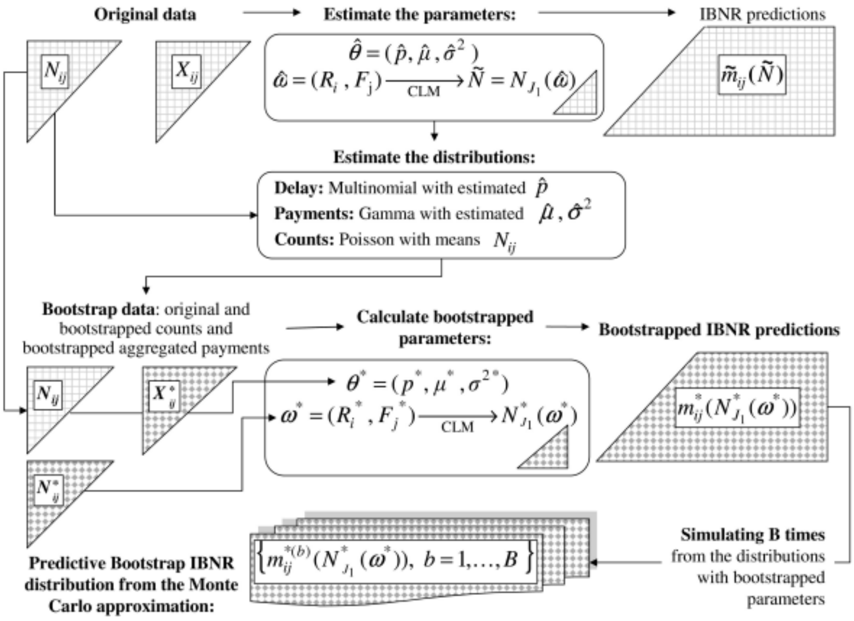

Fig. 3 Bootstrap algorithm for IBNR.

The steps shown in the Fig. 3 are similar to those already described in RBNS. This figure illustrates the flowchart of the steps of the bootstrap approach for IBNR claims. For IBNR, the bootstrap is not only applied to the claim amounts, but also to the claim counts. The original and bootstrapped claim counts and the bootstrapped claim amounts will both be used to derive the bootstrapped IBNR claim reserves. Further computation is as described in the RBNS algorithm.

3. Method of Analysis

3.1. Data

The data used in this research was a secondary data, namely claims data during the period of 2011–2020 obtained from one of the insurance companies in Indonesia. The claims were presented in two types of run-off triangle, incremental and cumulative respectively. The first run-off triangle presented the number of claims N_{ij} that occurred in accident period i and reported in $i + j$. The second run-off triangle presented the amount of claim payments X_{ij} that occurred in accident period i and period delay j.

Table 3. Incremental Run-off Triangle of Claim Counts

Accident Period (*i*)	Period Delay (*j*)									
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1	14214	2245	134	83	79	76	72	73	73	72
2	14347	1922	158	97	78	74	72	72	73	
3	14091	1869	169	91	77	73	73	73		
4	14005	1674	156	96	78	75	73			
5	13034	1575	137	83	74	74				
6	12298	1424	146	90	79					
7	11196	1419	129	84						
8	10432	1379	128							
9	10443	1213								
10	10507									

Table 3 presents the claim counts in the form of run-off triangle. For example, the total claim in the 3rd accident period in the first period delay is 14,091 claims. Table 4 given in the Appendix presents the claim data in the form of a run-off triangle and future triangle in incremental format for claims periods. This table displays the claims data that occurred during the development period of up to 10 years, allowing the observation of how claims develop over time. Each row in the table represents a specific claim year, and the columns show the accumulated claim amounts in the following years (development periods). For example, for claims that occurred in 4th period, the claims in the first year are 7.777.154, and so on up to the tenth year.

3.2. Research Procedure

The research process involved several stages for estimating and forecasting claim reserves. Initially, the claim counts and claim amounts data were presented in both incremental and cumulative forms of a run-off triangle. Then, the loss development factor value of the run-off triangle of claim counts $\hat{\lambda}_j$ dan and claim amounts $\tilde{\lambda}_j$ were estimated using the chain ladder method. Following this step, parameters for the run-off triangle of claim counts $(\hat{\alpha}_i, \hat{\beta}_j)$ and run-off triangle of claim amounts $(\tilde{\alpha}_i, \tilde{\beta}_j)$ were estimated. Subsequently, the RBNS delay parameters, namely $\hat{\pi}_l$ dan $\hat{p}_l$ and individual payment parameters, namely $\hat{\mu}$, $\hat{\gamma}_i$, dan $\hat{\sigma}^2$ were estimated. After that, the bootstrap approach was applied to the double chain ladder method for RBNS and IBNR claims. The flowchart of the steps is presented in Fig. 4.

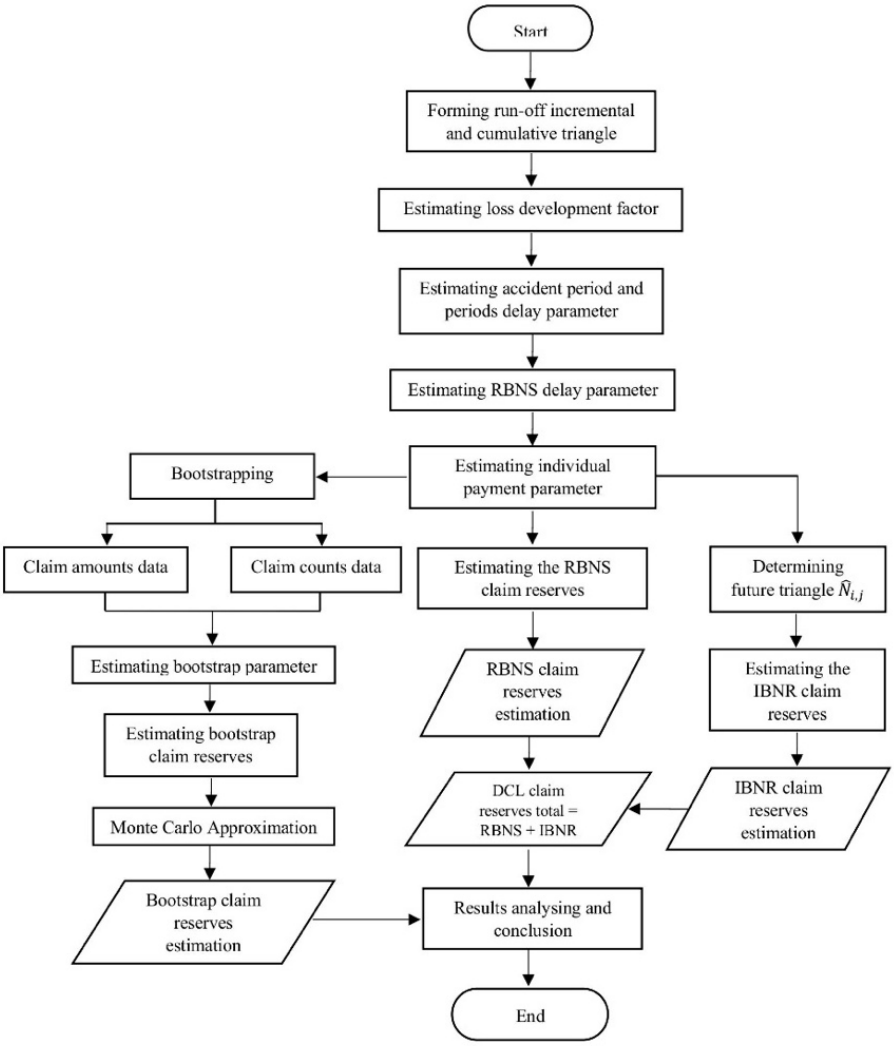

Fig. 4 Research flowchart.

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4. Result and Discussion

4.1. Claim Reserve Prediction with Double Chain Ladder Method

The estimation of claim reserves using the double chain ladder method was carried out by processing data on the run-off triangle of claim counts and claim amounts. Both of the run-off triangles were presented in incremental and cumulative. The incremental run-off triangle of claim counts in Table 2 and claim amounts in Table 3 were first transformed into a cumulative run-off triangle using (1).

The cumulative run-off triangle in Table 5 displays the cumulative number of claims in each accident period i and period delay j. The same steps were applied to generate the cumulative run-off triangle of large claims.

Table 5. Cumulative Run-off Triangle of Claim Counts

Accident Period (*i*)	Period Delay (*j*)									
	0	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***
1	14,214	16,459	16,593	16,676	16,755	16,831	16,903	16,976	17,049	17,121
2	14,347	16,269	16,427	16,524	16,602	16,676	16,748	16,820	16,893	
3	14,091	15,960	16,129	16,220	16,297	16,370	16,443	16,516		
4	14,005	15,679	15,835	15,931	16,009	16,084	16,157			
5	13,034	14,609	14,746	14,829	14,903	14,977				
6	12,298	13,722	13,868	13,958	14,037					
7	11,196	12,615	12,744	12,828						
8	10,432	11,811	11,939							
9	10,443	11,656								
10	10,507									

Table 6 given in the Appendix presents the cumulative claim amounts (in thousand) for each accident period i and period delay. The estimation of the double chain ladder method was initiated by calculating the loss development factor parameter using the chain ladder method on the run-off triangle of claim counts and claim amounts. The loss development factor in the run-off triangle of claim counts is denoted by $\hat{\lambda}_j$ and in the run-off triangle of claim amounts is denoted by $\tilde{\lambda}_j$. Table 7 provides the loss development factor estimation for claim counts and claim amounts as a whole.

Table 7. Loss Development Factor Claim Counts and Claim Amounts

j	$\hat{\lambda}_j$	$\tilde{\lambda}_j$
1	1.1290	1.5539
2	1.0098	1.0710
3	1.0058	1.0388
4	1.0049	1.0372
5	1.0046	1.0140
6	1.0043	1.0028
7	1.0043	1.0068
8	1.0043	1.0018
9	1.0042	1.0140

The loss development factor of claim counts $\hat{\lambda}_j$ in Table 7 shows a decreasing pattern for each period delay j, while the loss development factor of claims amounts $\tilde{\lambda}_j$ fluctuates. The loss development factor of claim counts at period delay $j = 1$ was 1,1290, suggesting that the number of claims settled when it is less than equal to the 1st j period delay $(j \leq 1)$ is about 1.1 times the number of claims settled at the 0th j period delay $(j = 0)$. Furthermore, it can be seen that the value of the loss development factor for many claims during the 9th period delay, $\hat{\lambda}_9$, is 1.0042 which tends to be close to 1. This is due to the fact that the cumulative claim counts at the corresponding period were not significantly different. The same understanding also applied to the loss development factor value

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of claim amounts. The loss development factor for claim amounts at the 1st period delay $\tilde{\lambda}_1$ of 1.5539 illustrates that claim amounts settled when it is less than or equal to the 1st period delay $(j \leq 1)$ is about 1.5 times the size of claim amounts settled at the 0th period delay $(j = 0)$. Simply put, the claim amounts settled at period delay $j \leq 1$ has a not too significant difference compared to the claim amounts settled without period delay.

4.2. Double Chain Ladder Parameter Estimation

The systematic calculation using the double chain ladder method was conducted to estimate all the parameters required, and then proceeded to estimate the RBNS and IBNR claim reserves. The first parameter to be estimated was $(\hat{\alpha}_i, \hat{\beta}_j)$ for the run-off triangle of claims counts and $(\tilde{\alpha}_i, \tilde{\beta}_j)$ for the run-off triangle of claims amounts. Table 8 shows that these parameters can be estimated by solving (7)–(12).

Table 8. Estimation of Parameters $(\hat{\alpha}_i, \hat{\beta}_j)$ for Claim Counts Run-off Triangle and $(\tilde{\alpha}_i, \tilde{\beta}_j)$ for Claim Amounts Run-off Triangle

i	$\hat{\alpha}_i$	$\tilde{\alpha}_i$	j	$\hat{\beta}_j$	$\tilde{\beta}_j$
1	17121.00	17,436,502.00	0	0.848869	0.536074
2	16964.34	16,678,331.19	1	0.109551	0.296967
3	16657.40	16,685,526.01	2	0.009468	0.059155
4	16366.24	13,572,747.58	3	0.005679	0.034704
5	15237.66	13,142,952.74	4	0.004809	0.034524
6	14347.24	11,473,152.91	5	0.004517	0.013490
7	13176.29	11,030,620.54	6	0.004321	0.002755
8	12335.11	10,874,071.01	7	0.004296	0.006713
9	12161.68	11,615,115.22	8	0.004283	0.001794
10	12377.64	11,399,207.11	9	0.004205	0.013824

Furthermore, the RBNS delay parameters, namely $\hat{\pi}_k$ and $\hat{p}_k$, were estimated. Parameter $\hat{\pi}_k$ denotes the proportion of claims settled and $\hat{p}_k$ denotes the probability of delay. Parameter $\hat{\pi}_k$ was derived by solving the system of linear in (13). Subsequently, $\hat{p}_k$ for $k = 0, 1, \dots, d$ was estimated. The value of d as the maximum delay parameter is $d = 6$. The estimation of $\hat{p}_k$ for $delay\ k = 0, 1, \dots, 5$ was obtained using (14). Afterward, the estimation of $\hat{p}_d$ for $delay\ k = d = 6$ was obtained using (15). The remaining $\hat{p}_k$ were set as 0 so that $\hat{p}_7 = \hat{p}_8 = \hat{p}_9 = 0$ was derived and the condition $\sum_{l=0}^{m-1} p_k = 1$ was satisfied. Table 9 present the estimated values of $\hat{\pi}_k$ and $\hat{p}_k$ for $delay\ k = 0, \dots, 9$.

Table 9. Estimation of RBNS Delay Parameter

k	$\hat{\pi}_k$	$\hat{p}_k$
0	0.6315	0.6315
1	0.2683	0.2683
2	0.0280	0.0280
3	0.0300	0.0300
4	0.0311	0.0311
5	0.0065	0.0065
6	-0.0029	0.0045
7	0.0031	0.0000
8	-0.0033	0.0000
9	0.0017	0.0000

The highest $\hat{p}_k$ occurred at delay $k = 0$, which was 0.6315. It indicated that claims count settled in the same year as the claim was reported have the largest probability proportion compared to other delays. About 63.15% of claims were settled in the same year as the year the claim was reported to the insurance company. Table 8 shows that the longer the delay occurs, the more likely the delay probabilities will be close to zero. It can be seen from $\hat{p}_k$ for $delay\ k = 6$ that is close to zero and

p-ISSN 2798-253X
e-ISSN 2798-3153

equal to zero for delay $k = 7, 8, 9$. It means that the proportion of claims settled within the 6-year delay period is very small and there are no claims settled within the 7-to-9-year delay period since the claim was reported.

The parameters that were estimated from the individual payment variable were the mean factor $\hat{\mu}$ and the inflation parameter $\hat{\gamma}_i$. Firs, the mean estimation $\hat{\mu}$ was obtained using (16). The mean estimation $\hat{\mu}$ was then used to determine the inflation parameter estimation $\hat{\gamma}_i$ using (17). Table 10 shows that the average value is 1,018.4277.

Table 10. Estimation of Individual Payment Parameter

i	$\hat{\gamma}_i$
1	1.0000
2	0.9654
3	0.9836
4	0.8143
5	0.8469
6	0.7852
7	0.8220
8	0.8656
9	0.9378
10	0.9043
$\hat{\mu}$ = 1018.4277	

It indicates the average of the individual claim payments for the first accident period that is $i = 1$. The estimation of the inflation parameter $\hat{\gamma}_i$ presented in Table 9 refers to the claim inflation for each accident period *i*. The estimation of $\hat{\gamma}_i$ for the accident period $i = 1$ derived from the calculation is 1. To prevent over-parametrised and the parameter can be identified easily, $\hat{\gamma}_i$ for the accident period is set to 1, so that $\hat{\gamma}_1 = 1$.

4.3. Calculation of RBNS and IBNR Claims Reserve Estimation with Double Chain Ladder Method

The estimation of claim reserves was carried out separately, i.e. RBNS and IBNR claim reserves. The RBNS claims reserve estimation requires the original data of claim counts, N_{ij}, in the upper triangle of index I and parameter $\hat{\theta} = (\hat{p}_l, \hat{\mu}, \hat{\gamma})$. The point forecast of the RBNS, $\hat{X}_{i,j}^{RBNS}$, was derived using (18). The point forecast of the RBNS component will vary along the index $J_1 \cup J_2$. The estimation of RBNS claim reserves based on the future year was determined by summing the point forecasts along the diagonal of the index $J_1 \cup J_2$; the value is on Table 11 given in the Appendix. In contrast to RBNS, the estimation of IBNR claim reserves requires data on claim counts in the future triangle $\hat{N}_{i,j}$, $(i, j) \in J_1$. Claim counts of future triangle claims, $\hat{N}_{i,j}$,was derived using the chain ladder method in (3), which is presented in Table 11.

The estimation future triangle for claim counts is illustrated in Table 12.

Table 12. Estimation Future of Triangle $\hat{N}_{i,j}$

Accident Period (*i*)	Period Delay (*j*)									
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1	14,214	16,459	16,593	16,676	16,755	16,831	16,903	16,976	17,049	17,121
2	14,347	16,269	16,427	16,524	16,602	16,676	16,748	16,820	16,893	16,964
3	14,091	15,960	16,129	16,220	16,297	16,370	16,443	16,516	16,587	16,657
4	14,005	15,679	15,835	15,931	16,009	16,084	16,157	16,227	16,297	16,366
5	13,034	14,609	14,746	14,829	14,903	14,977	15,043	15,108	15,174	15,238
6	12,298	13,722	13,868	13,958	14,037	14,102	14,164	14,225	14,287	14,347

p-ISSN 2798-253X
e-ISSN 2798-3153

7	11,196	12,615	12,744	12,828	12,891	12,951	13,008	13,064	13,121	13,176
8	10,432	11,811	11,939	12,009	12,068	12,124	12,177	12,230	12,283	12,335
9	10,443	11,656	11,771	11,840	11,899	11,954	12,006	12,058	12,111	12,162
10	10,507	11,863	11,980	12,050	12,110	12,166	12,219	12,273	12,326	12,378

Once the data of claim counts in the future triangle was obtained, the point prediction of the IBNR component, $\hat{X}_{i,j}^{IBNR}$ was determined to satisfy index $J_1 \cup J_2 \cup J_3$ using (19 The sum of the point predictions along the diagonal of those indices was used to estimate the IBNR claim reserves for the future year. Table 13 presents the prediction of RBNS, IBNR, and total claim reserves using the double-chain ladder method.

Table 13. Prediction of RBNS and IBNR Claim Reserve Based on Future Year

Future Years	RBNS (IDR)	IBNR (IDR)	Total (IDR)
1	4,203,868	1,128,297	5,332,165
2	1,135,437	802,551	1,937,988
3	829,267	427,970	1,257,238
4	481,297	364,054	845,350
5	127,015	318,391	445,407
6	51,571	234,782	286,354
7	0	182,424	182,424
8	0	127,760	127,760
9	0	77,780	77,780
10	0	28,948	28,948
11	0	11,142	11,142
12	0	6,295	6,295
13	0	2,781	2,781
14	0	752	752
15	0	216	216
16	0	0	0
17	0	0	0
18	0	0	0
Total	**6,828,456**	**3,714,144**	**10,542,600**

Thus, it is assumed that the estimation results of the double chain ladder claim reserves varies according to the index $J_1 \cup J_2 \cup J_3$.

4.4. Predictive Distribution with Bootstrap Approach on Double Chain Ladder

The bootstrap approach was applied to determine the predictive distribution of the estimated claim reserves that had been derived. The predictive distribution arising from the bootstrap approach consists of mean, percentile Q_n, and prediction error (p.e) of each claim reserve. In this study, the bootstrap approach was applied by taking into account the possibility of uncertainty in the double chain ladder parameters. The bootstrap technique was performed with the help of Monte Carlo approximation with 1,000 iterations. Table 14 presents the predictive distribution results of RBNS, IBNR, and total claim reserves using the double chain ladder with a bootstrap approach

Table 14. Predictive Distribution of DCL Claim Reserves with Bootstrap Approach

Predictive Distribution	RBNS (IDR)	IBNR (IDR)	Total (IDR)
mean	6,777,539	3,741,979	10,519,518
p.e	1,277,450	679,300	1,956,750
1%	4,369,807	2,351,784	6,721,591
5%	4,905,282	2,772,513	7,677,795
50%	6,626,822	3,656,721	10,283,543
95%	9,094,092	4,996,680	14,090,772

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99%	10,329,589	5,485,124	15,814,713

As shown in Table 14, the mean of RBNS claim reserves with the bootstrap approach is IDR6,777,539,000, and IBNR is IDR3,741,979,000, so the total claim reserves obtained are IDR 10,519,518,000. The prediction error of the claim reserve is IDR1,956,750,000. It indicates that the prediction error is 14.94% of the total claim reserves. For the 1st percentile, the amount of claims reserves earned was IDR6,721,591,000. This number indicates that the chance of total claims to be less than IDR6,721,591,000 is 1%. For the 95th percentile, the amount of claim reserves was IDR14,090,772,000, meaning that the probability of total claims to be less than Rp 14,090,772,000 is 95%. If the company decides to set aside a claim reserve fund of IDR15,814,713,000, with the expectation that the chance of default is 1%, the company must have an adequate financial condition to ensure that this amount is accomplished. Otherwise, the company would incur more funds to obtain additional allocation. On the other hand, if the company sets aside a claim reserve fund allocation amounting to IDR14,090,772,000, it must be prepared to handle a greater chance of default risk, which is 5%. However, given this amount, the company does not necessarily need to set aside a large claim reserve fund. Instead, the company can allocate funds to other financial instruments, such as investments, to earn profits.

5. Conclusion

Estimation using the double chain ladder method only generates the estimation of the claim reserves without providing any information about the distributional properties of the claim reserves. Based on the calculation, the claim reserve estimation using the double chain ladder method, , the estimated value of the RBNS claim reserve was IDR6,828,456,000 and the estimated IBNR claim reserve was IDR3,714,144,000. The total estimated claim reserve value obtained was IDR10,542,600,000. Applying the bootstrap approach to the double chain ladder, the predictive distribution was derived in the form of the mean, prediction error, and percentile of the claim reserves. Results showed that the mean of RBNS claim reserves with the bootstrap approach was IDR6,777,539,000 and IBNR was IDR3,741,979,000, so the total claim reserves obtained were IDR10,519,518,000. In can be concluded that the greater the nominal claim reserve allocated, the lower the chance of the company going bankrupt. The information about predictive distribution, especially prediction error, allows insurance companies to evaluate the mean and percentile value of claim reserves and predict the development of a claim in the future so that the company is able to set the appropriate amount of claim reserves to prevent insolvency.

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Appendix

Table 4. Incremental Run-off Triangle of Claim Amounts

Accident Period (*i*)	Period Delay (*j*)									
	0	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***
1	8,848,190	4,748,588	1,390,771	1,140,682	412,162	360,063	20,241	220,299	54,467	241,039
2	9,070.763	5,890,750	519,880	539,274	127,773	86,544	122,132	83,925	6,732	
3	8,763,326	4,293,516	1,339,468	292,402	1,515,687	155,474	28,282	36,781		
4	7,777,154	4,145,306	642,888	504,199	92,102	101,322	6,692			
5	7,213,056	3,498,302	778,204	354,927	626,514	342,254				
6	6,265,529	3,737,703	546,716	182,562	298,067					
7	5,737,519	3,281,541	748,174	457,055						
8	5,612,304	3,495,658	593,846							
9	6,386,096	3,289,775								
10	6,110,822									

Table 6. Cumulative Run-off Triangle of Claim Amounts

Accident Period (*i*)	Period delay (*j*)									
	0	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***
1	8,848,190	13,596.778	14,987,549	16,128,231	16,540,393	16,900,456	16,920,697	17,140,996	17,195,463	17,436,502
2	9,070,763	14,961.513	15,481,393	16,020,667	16,148,440	16,234,984	16,357,116	16,441,041	16,447,773	
3	8,763,326	13,056.842	14,396,310	14,688,712	16,204,399	16,359,873	16,388,155	16,424,936		
4	7,777,154	11,922.460	12,565,348	13,069,547	13,161,649	13,262,971	13,269,663			
5	7,213,056	10,711.358	11,489,562	11,844,489	12,471,003	12,813,257				
6	6,265,529	10,003.232	10,549,948	10,732,510	11,030,577					
7	5,737,519	9,019.060	9,767,234	10,224,289						
8	5,612,304	9,107.962	9,701,808							
9	6,386,096	9,675.871								
10	6,110,822									

Table 11. Estimation Future Triangle $\hat{X}_{i,j}$

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e-ISSN 2798-3153

Accident Period (*i*)	**Period delay (*j*)**									
	0	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***
1	8,848,190	13,596,778	14,987,549	16,128,231	16,540,393	16,900,456	16,920,697	17,140,996	17,195,463	17,436,502
2	9,070,763	14,961,513	15,481,393	16,020,667	16,148,440	16,234,984	16,357,116	16,441,041	16,447,773	16,678,042
3	8,763,326	13,056,842	14,396,310	14,688,712	16,204,399	16,359,873	16,388,155	16,424,936	16,454,501	16,684,864
4	7,777,154	11,922,460	12,565,348	13,069,547	13,161,649	13,262,971	13,269,663	13,359,897	13,383,945	13,571,320
5	7,213,056	10,711,358	11,489,562	11,844,489	12,471,003	12,813,257	12,849,134	12,936,508	12,959,794	13,141,231
6	6,265,529	10,003,232	10,549,948	10,732,510	11,030,577	11,185,005	11,216,323	11,292,594	11,312,921	11,471,302
7	5,737,519	9,019,060	9,767,234	10,224,289	10,604,633	10,753,097	10,783,206	10,856,532	10,876,074	11,028,339
8	5,612,304	9,107,962	9,701,808	10,078,238	10,453,149	10,599,493	10,629,171	10,701,450	10,720,712	10,870,802
9	6,386,096	9,675,871	10,362,858	10,764,937	11,165,392	11,321,708	11,353,409	11,430,612	11,451,187	11,611,504
10	6,110,822	9,495,606	10,169,794	10,564,382	10,957,377	11,110,781	11,141,891	11,217,656	11,237,848	11,395,177

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