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Pension Funding Calculation Using Benefit Prorate Method and Vasicek Model-Based Interest Rates

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ABSTRACT

Pension funds are financial programs established by individuals or companies to secure the employees' retirement years by providing them benefits. Accurate interest rate predictions lead to more precise calculations of pension fund contributions. Pension funds are built up through contributions made by both the participants (employees) and the employer. Various methods were used to calculate the amount of pension benefits, normal costs, and actuarial liabilities, including the constant percent benefit prorate method. A key factor influencing these calculations is the interest rate. This study employed the Vasicek model, a stochastic interest rate model, to analyze the unfunded actuarial liability (UAL) using historical BI-Rate data. The Vasicek model is deemed acceptable for interest rate forecasting, even with a 40.9% mean absolute percentage error (MAPE). The analysis revealed that the normal costs and actuarial liabilities for each participants will adjust based on the interest rate during their retirement period. The amount of UAL is derived from the discrepancy between the total amount of actuarial liabilities from all the participant and the accumulated funds. The UAL is IDR10,108,184,176, sufficient to cover future pension fund payments and there will be profit received from the pension fund program.

1. Introduction

A pension fund program is a system designed to provide individuals with income during their retirement years [1]. It is essentially a way to save and invest money during a person's career to ensure financial security after retirement. Pension funds are typically managed by financial institutions or government agencies. Pension programs are government-mandated schemes designed to ensure financial security in retirement year, as outlined in Law Number 11 of 1992 related to Pension Funds [2]. Pension programs come in two main types: defined benefit and defined contribution. Defined benefit plans specify a fixed benefit amount, while defined contribution plans have predetermined regular contributions [3].

Defined benefit pension program can be calculated using various methods, such as accrued benefit method, benefit prorate method, and cost prorate method [4]. The benefit prorate method in pension fund calculations is a way to allocate pension benefits based on proportion of an employee's service rendered up to a certain point in time. The method evenly distributes pension benefits over time with two approaches to calculate normal costs and actuarial liability: constant dollar and constant percent [5]. The constant dollar approach considers years of service until retirement, while the constant percent approach is based on the participant's salary or wage until retirement [6]. The normal costs that are calculated using the benefit prorate method of the constant percent type will vary each year due to salary increases.

Pension funds are accumulated through contributions from both pension program participants (employees) and employers [7]. These funds are invested to generate returns that can cover future pension obligations. A pension fund deficiency occurs when the projected future liability exceeds the current accumulated fund balance. Several factors influence pension calculations, including mortality rates, survival rates, and interest rates. Interest plays a crucial role in projecting future fund growth and determining the size of pension annuities [8]. For the purpose of understanding and modeling interest rate dynamics, The Vasicek model provides a mathematical description of their evolutionary path. The Vasicek model is stochastic interest rate model that predicts future interest rates based on the previous year's interest rate (past trends) and a mean reversion property, meaning it tends to move toward an equilibrium point [9].

This study focuses on analyzing the unfunded actuarial liability (UAL) within a defined benefit pension program using the constant percent benefit prorate method and the Vasicek interest rate model. The benefit prorate method was chosen because pension benefits are allocated evenly each year based on participants' salaries, making it easier to calculate normal cost and actuarial liability. In addition, the Vasicek interest rate model accurately reflects future interest rates in fluctuating market conditions. An improved evaluation of pension fund adequacy, along with the identification of potential funding deficiencies, was sought in this study by integrating interest rate based on the Vasicek model into UAL calculations using the benefit prorate method.

2. Defined Benefit Pension Plans

Participants salaries are determined by their years of service and salary increases. Salary increments are calculated by analyzing the relationship between participant's incomes, service durations, and ages within a specific year, establishing average income growth patterns. This average income growth patterns is then formalized into a salary scale, which enables the prediction of future participant income. The cumulative salary up to age x years is denoted as S_x dan and expressed as follows [10]:

$$S_x = \sum_{t=e}^{x-1} s_t \text{ , 9} \quad (1)$$

the entry age is denoted by e and s_t is the participant's annual salary at specified age t years. If the salaryat age x years is subject to an annual increase of s, then the salary at age $x + t$ years can be determined by:

$$s_{x+t} = s_x(1 + s)^t. \quad (2)$$

The salary of the participant at age x years is denoted by s_x. The salary function is used to calculate the value of the benefits for pension fund participants.

Benefit function is used to determine the amount of pension benefits that participants will receive upon reaching retirement age [10]. The pension benefit amount is denoted by B_r. To calculate pension benefits, the average salary earned throughout the employee's career is considered.The final pension amount is determined based on a specific proportion of either the salary at a certain age x years or the cumulative salary up to that age [1]. The total pension benefits accumulated up to retirement age at r years are defined based on an α proportion of the cumulative salary earned until that age is calculated using (3).

$$B_r = \alpha S_r. \quad (3)$$

A whole life annuity-due refers to a series of payments made at the beginning of each period, continuing as long as the insured or pension program participant is alive [11]. The payments are typically made annually and can be expressed as in (4) [12].

$$\ddot{a}_x = 1 + vp_x + v^2{}_2p_x + v^3{}_3p_x + \cdots = \sum_{k=0}^{\infty} v^k{}_kp_x. \tag{4}$$

A defined benefit pension plan is a retirement plan, which an employer promises a spesific payment to employees upon their retirement. This plan considers years of service with the company, the employee's final salary, and age at retirement. The present value of future benefits (PVFB) represents the current value of the pension benefits that will be paid to a participant in the future, starting from their retirement age until their death [13]. Equation (5) is used for calculating PVFB.

$$PVFB = B_r \ddot{a}_r v^{r-x}{}_{r-x}p_x\ . \tag{5}$$

Normal cost (NC) refers to the regular contributions or amount of money that must be paid by active employees to fund their future pension benefits. When using the benefit prorate method with a constant percent type, the normal costs is calculated based on a proportion of the current salry (s_x) compared to the total salary earned until retirement (S_r) [14]. Equation (6) is usef for calculating normal costs using the benefit prorate method of the constant percent type.

$$(NC)_x = \frac{s_x}{S_r}(PVFB)_x\ . \tag{6}$$

The aggregate fund accumulation in a given year t is derived by summing the annual normal costs payment contributed by all N participants, represented by the value of TNC_t.This involves summing up the individual normal costs of each participant, as shown below:

$$TNC_t = \sum_{j=1}^{N} NC_{x_j}\ . \tag{7}$$

Actuarial liability (AL) refers to the funds that a pension fund company must set aside to guarantee the payment of future pension benefits to participants of the pension program [15]. When using the benefit prorate method with a constant percent type, actuarial liabilityis calculated based on the proportion of cumulativesalary earned up to the current age compared to the total cumulative salary until retirement. The total actuarial liability for group consists of Nparticipants of the same age can be obtained by adding up the individual actuarial liability of each participant in that group.

$$AL_t = \sum_{j=1}^{N} \frac{S_{x_j}}{S_r}(PVFB)_{x_j}\ . \tag{8}$$

UAL indicates wheter the pension fund has sufficient amount to cover its future liabilities to their participants and is expressed (9) [16]:

$$UAL_t = TAL_t - F_t\ , \tag{9}$$

the total funds accumulated at the end of the year t, denoted by F_t, can be calculated using (10) [17].

$$F_t = F_{t-1} + TNC_t + IR_t - TAB_t. \tag{10}$$

The benefits distrubuted to participants due to death or resignation during year t are denoted as TAB_t. The total investment return earned in year$t$, denoted as IR_t, can be obtained using (11).

$$IR_t = (F_{t-1} \times i) + \left(TNC_t \times \frac{i}{2}\right) - (TAB_t \times i). \tag{11}$$

The investment return rate (i) is assumed to be equal to the interest rate. The the UAL can be either positive or negative. A positive UAL means that the accumulated funds are insufficient to cover future actuarial liabilities; while a negative UAL indicates that the accumulated funds exceed the actuarial liabilities. This assessment helps determine the adequacy of the pension funds.

3. Vasicek Model

The Vasicek interest rate model is characterized by a mean-reverting stochastic process, wherein interest rates exhibit fluctuations around a long-term equilibrium level [18].The Vasicek model is characterized by three parameters, such as β, θ, and σ. The parameter β determines how quickly the

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short-term interest rate $r(t)$ reverts to its long-term mean. The parameter θ represents this long-term mean, while parameter σ signifies the volatility of the short-term interest rate. Suppose $W(t)$ denotes a Wiener process, based on the properties of Itô's integral, the interest rate for each period can be predicted using (12) [19]:

$$r(t_i) = r(t_{i-1})e^{-\beta(\Delta t)} + \theta\left(1 - e^{-\beta(\Delta t)}\right) + \sigma e^{-\beta t}\int_{t_{i-1}}^{t_i} e^{\beta u} dW(u). \tag{12}$$

The parameters of the Vasicek model are estinated through maximum likelihood estimation (MLE), a technique that seeks to maximize the likelihood of the observed data [21]. Hence, the log-likelihood function based on the probability density function is as follows:

$$\ln L(r(t_i);\beta,\theta,\sigma) = \ln(2\pi)^{-\frac{n}{2}} + \ln(\sigma^2)^{-\frac{n}{2}} - \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(r(t_i) - r(t_{i-1})e^{-\beta\Delta t} - \theta\left(1 - e^{-\beta\Delta t}\right)\right)^2.$$

The log-likelihood function is maximized through differentiation, solving for each parameter. The resulting parameter estimates are then calculated using the equations listed following:

$$\hat{\beta} = -\frac{1}{\Delta t} \times \left(\ln \frac{\sum_{i=1}^{n}\left(r(t_i)r(t_{i-1}) - \theta\sum_{i=1}^{n} r(t_i) - \theta\sum_{i=1}^{n} r(t_{i-1}) + n\theta^2\right)}{\sum_{i=1}^{n}\left(r(t_{i-1})\right)^2 - 2\theta\sum_{i=1}^{n} r(t_{i-1}) + n\theta^2}\right) \tag{13}$$

$$\hat{\theta} = \frac{\sum_{i=1}^{n}\left(r(t_i) - r(t_{i-1})e^{-\beta\Delta t}\right)}{n\left(1 - e^{-\beta\Delta t}\right)} \tag{14}$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\Big[r(t_i)^2 - 2e^{-\beta\Delta t}r(t_i)r(t_{i-1}) + e^{-2\beta\Delta t}r(t_{i-1})^2 - 2\theta\left(1 - e^{-\beta\Delta t}\right)\left(r(t_i) - e^{-\beta\Delta t}r(t_{i-1})\right) + \theta^2\left(1 - e^{-\beta\Delta t}\right)^2\Big]. \tag{16}$$

The Vasicek model was used to forecast interest rates that were used in calculating normal costs NC, AL, and the UAL for pension fund based on the benefit prorate method, as shown in the flowchart in Fig. 1.

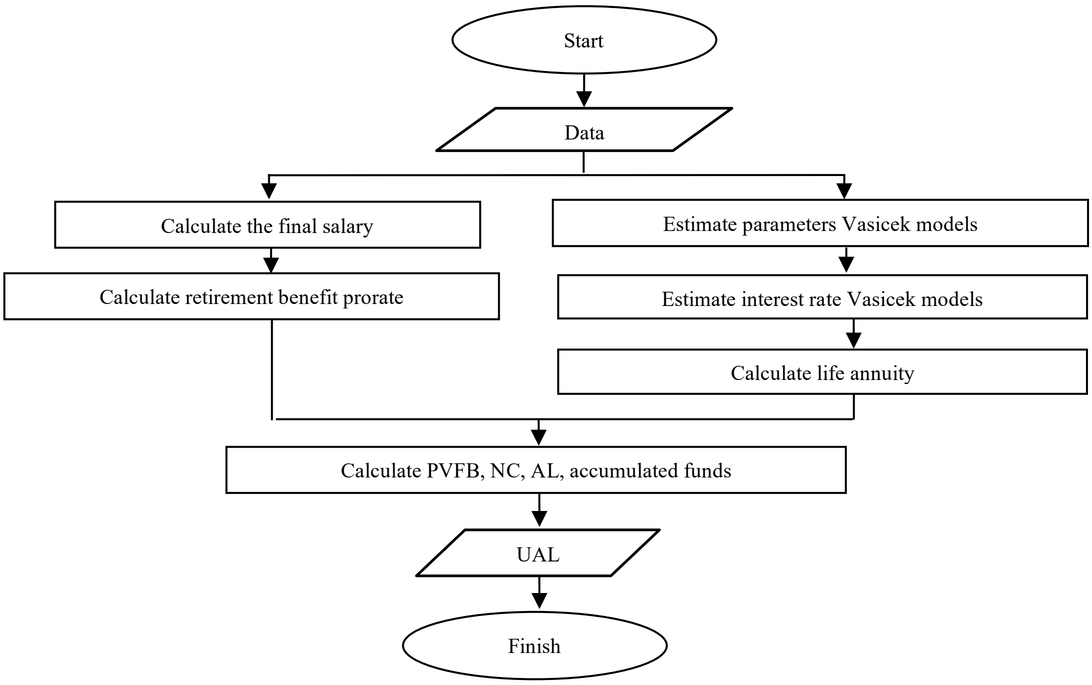

Fig. 1 Flowchart pension funding calculation using benefit prorate and Vasicek model.

4. Results and Discussion

The initial phase in applying the Vasicek model to interest rate determination involves parameter estimation. The parameters $\beta, \theta,$ and σ were estimated using historical BI-Rate data, and the resulting values are presented in Table 1.

Table 1. Parameters of the Vasicek Model

Parameter	Value
$\hat{\beta}$	0.1045
$\hat{\theta}$	0.5316
$\hat{\sigma}$	0.0521

The parameter $\hat{\beta}$ indicates that the interest rate will return to the mean at a rate of 0.1045; $\hat{\theta}$ interprets the average interest rate to be 0.5316; and $\hat{\sigma}$ indicates the fluctuation of the interest rate is 0.0521. Hence, the interest rate equation can be derived using the parameter's value shown in Table 2, such as:

$$r(t_i) = r(t_{i-1})e^{-0,1045} + 0,5316(1 - e^{-0,1045}) + 0,0521e^{-0,1045t}\int_{t_{i-1}}^{t_i} e^{0,1045u}dW(u)\,. \quad (17)$$

Equation (17) was used to generated Vasicek model-based interest rate predictions using historical BI-Rate data from 1985 to 2023, beginning with an initial interest rate of 13.79%. The accuracy of these predictions was evaluated using the MAPE. The model demonstrated suitability for interest rate forecasting, with a MAPE of 40.9%. The model is suitable for estimating the interest rate of the Vasicek model since the MAPE value is still below 50%. This model was then employed to forecast annual interest rates for the next 20 periods, from 2024 to 2044, using an initial interest rate of 6%. The result of these Vasicek interest rate estimations is presented in Fig. 2.

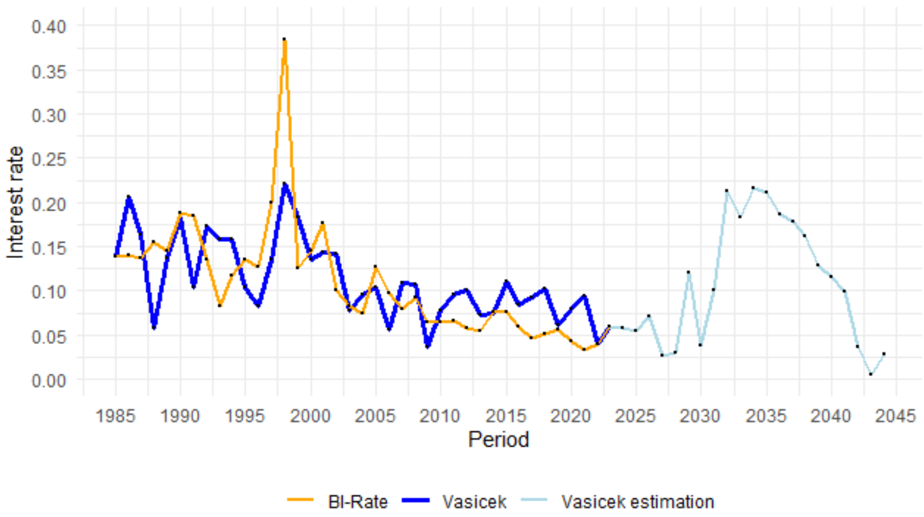

Fig. 2 Interest rate forecasting using Vasicek model.

Fig. 2 shown the actual BI-Rate interest rate during 1985–2023 and the estimated Vasicek model-based interest rates during 1985–2044. The estimated Vasicek model-based interest rates fluctuate during 2024–2044, showing a downward trend from 2034, with the highest estimated rate occurring in 2032.

3.1. Defined Benefit Pension Plans Using Vasicek Model Based-Interest Rates

The forecasted interest rates graph in Fig. 2 exhibits fluctuations and a downward trend after 2034. These forecasted interest rates form the basis for determining the whole life annuity-due upon retirement year, detailed in Table 2 for 2024–2044.

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Table 2. Whole Life Annuity-Due

Retirement year	Whole Life Annuity
2024	12.58
2025	12.92
2026	11.16
.	.
.	.
.	.
2042	15.41
2043	22.81
2044	17.07

Fluctuations in annual interest rates directly impact annuity values through their inverse relationship with discount factors. An increase in interest rates lower discount factors, thereby reducing annuity amounts. Pension benefits were estimated based on a participant's salary, assuming an 8% annual growth rate, with 2.5% of the salary allocated to pension. PVFB was then calculated, as summarized in Table 3.

Table 3. Present Value of Future Benefits (PVFB)

Entry Age	Current Age	Current Salary	Final Cumulative Salary	Pension Benefits	PVFB
18	56	IDR58,695,581	IDR816,389,143	IDR20,409,728	IDR193,906,676
18	56	IDR48,213,377	IDR670,593,548	IDR16,764,836	IDR159,277,677
18	55	IDR46,740,914	IDR699,412,002	IDR17,485,300	IDR268,841,245
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
26	42	IDR33,215,806	IDR1,301,250,866	IDR32,531,271	IDR41,904,630
26	41	IDR32,202,438	IDR1,362,475,621	IDR34,061,890	IDR57,483,482
27	44	IDR42,589,463	IDR1,419,786,122	IDR35,494,653	IDR25,769,761
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
48	58	IDR0	IDR198,893,679	IDR4,972,341	IDR62,562,400
49	59	IDR0	IDR278,873,670	IDR6,971,841	IDR0
49	59	IDR0	IDR176,848,297	IDR4,421,207	IDR0

The final cumulative salary, pension benefits, and PVFB are directly proportional to the annual salary and service durations. Participants who have exceeded the retirement age are entitled to their pension benefits, resulting in a zero PVFB. The calculated PVFB was then used to determine the normal costs and actuarial liabilities of pension plan participants. In the constant percent prorated method, the prorated salary used to calculate the normal contribution is derived from the salary at the age of calculation (s_x) divided by the cumulative salary until retirement age (S_r).

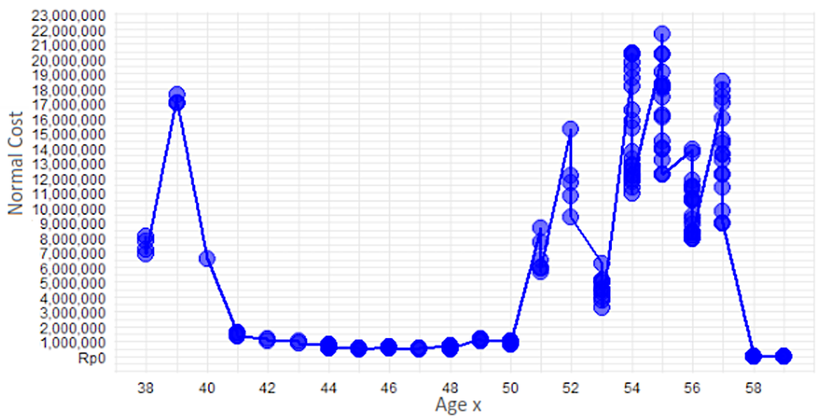

Fig. 1 Normal contributions for all participants.

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Fig. 3 illustrates the participants aged 38 to 40 experienced higher contribution rates in 2024 due to projected lower interest rates at retirement. Retired participants were not subject to prorated salary calculations, resulting in zero normal contributions. The total normal cotribution for 2024 were calculated to determine the accumulated funds. The total normal contribtutions for 2024 were calculated, amounting to IDR1,293,224,629, with 29 participants reaching retirement in that year.

Following the determination of the normal contribution, the AL was calculated. Using the constant percent prorate method, the prorated salary needed for this calculation was determined by dividing the cumulative salary at the current age(S_x) by the cumulative salary at retirement (S_r).

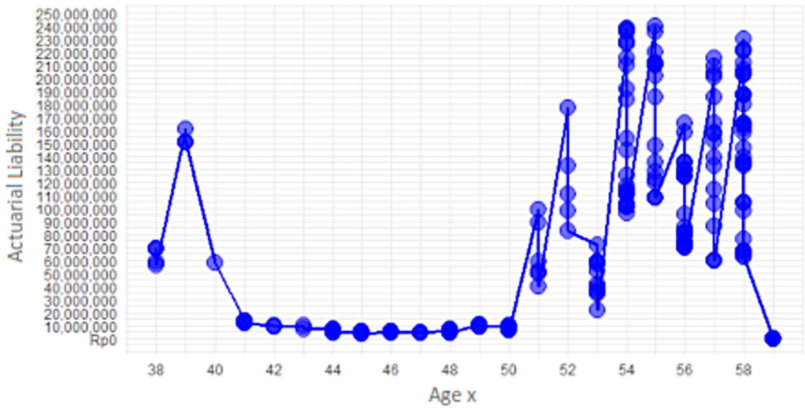

Fig. 4 Actuarial liability for all participants.

Upon reaching retirement age, participants become entitled to pension benefits, due to the AL being non-zero. After retirement, AL decrease to zero as all pension benefits have been disbursed. Next, to determine the UAL, the total actuarial liability for 2024 was calculated (Fig. 4). The total of AL for 2024 amounted to IDR17,768,616,418.

The number of retirees increased significantly, from 6 persons in 2023 to 29 persons in 2024. Consequently, the pension benefits for retired participants in 2023 (TAB_{2023}) was amounted to IDR70,663,366 and in 2024 (TAB_{2024}) is amounted to IDR514.257.957. In the UAL calculation, a 5.77% investment returns assumed, reflecting the 2024 interest rate. The accumulated funds from during 1985-2024 using the Vasicek interest rate estimations are shown in Table 5.

Table 5. Accumulated Funds

Year	Interest Rate	Total Normal Costs	Total Pension Benefits for Retired Participants	Investment	Accumulated Value
1985	13.79%	IDR0	IDR0	IDR0	IDR0
1986	21%	IDR544,522	IDR0	IDR56,152	IDR600,674
1987	16%	IDR1,274,274	IDR0	IDR203,029	IDR2,077,977
1988	6%	IDR2,618,505	IDR0	IDR196,704	IDR4,893,187
1989	14%	IDR5,937,771.34	IDR0	IDR1,080,770	IDR11,911,729
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
2004	9.61%	IDR84,790,550	IDR0	IDR124,434,144	IDR1,461,059,979
2005	10.42%	IDR95,643,610	IDR0	IDR157,280,733	IDR1,713,984,324
2006	5.69%	IDR114,930,022	IDR0	IDR100,874,181	IDR1,929,788,528
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
2021	9%	IDR1,211,230,741	IDR0	IDR1,710,037,767	IDR20,405,899,835
2022	4%	IDR1,382,036,087	IDR0	IDR835,491,020	IDR22,623,426,942
2023	6%	IDR1,493,750,402	IDR70,663,366	IDR1,397,978,326	IDR25,444,492,304
2024	5.77%	IDR1,293,224,629	IDR514,257,957	IDR1,487,454,245	IDR27,876,800,594

The accumulated fund in 1985 is IDR0 because based on the data obtained, the pension program started in 1986. Total normal costs generally rise each year, however, in 2024, a reduction occurred because 29 participants reached retirement age so they stopped paying normal costs. The investment rate fluctuates with the prevailing interest rate. Changes in normal costs and accumulated value during 1985–2024, as effected by the Vasicek model-based interest rates shown in Fig. 5.

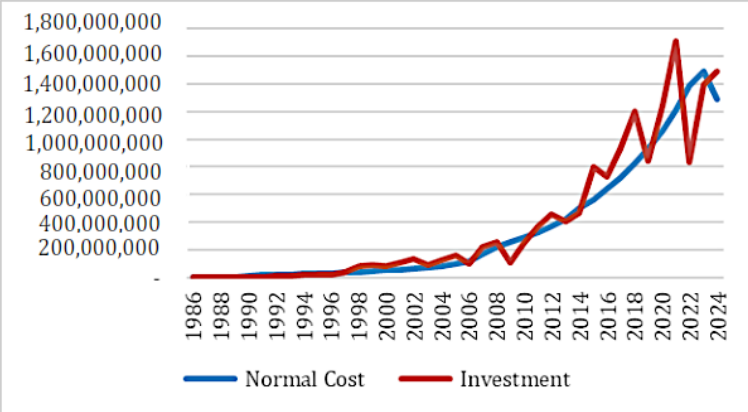

Fig. 5 Trends in contributions and investments.

The accumulated funds in 2024 and the total actuarial liability in that year are used to calculate the UAL. The calculation results show that the UAL in 2024 was sufficient to finance the payment of retirement benefits to participants, with a surplus of IDR10,108,184,176; indicating that the pension fund is stable and able to fulfill the requirement to pay benefits in the future.

5. Conclusion

Using the Vasicek interest rate model, conclude that higher interest rates at the time of retirement lead to smaller pension benefits for participants. An increase in the interest rate by 2% correlates with a 3%–5% increase in normal costs and a corresponding decrease in the present value of future pension benefits, thereby lowering the required reserve fund. The unfunded actuarial liability using the Vasicek interest rate shows a significant surplus, indicating that the pension fund is well-positioned to meet its future liabilities. The fluctuating nature of Vasicek interest rate aligns with real-world economic conditions in Indonesia, enabling the company to make more informed financial decisions. For more consistent-interest rate estimation with less fluctuation, the Cox-Ingersoll-Ross (CIR) model can be employed. The CIR model is ensuring that interest rates remain non-negative.

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