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Research Article

Forecasting of Export Value in Indonesia Using Top-Down Hierarchical Time Series Based on Historical Proportion

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Abstract: Export is a trading activity carried out between countries by bringing or sending goods originating from within the country to foreign countries with the aim of selling or marketing them. Exports as a source of state revenue are needed for the economy because they can make a major contribution to economic stability and growth. Export values that experience a decrease or increase in the future need to be considered. For this reason, the purpose of this study is to forecast the value of exports in Indonesia for the future period. Export value data is treated as hierarchical time series data. The top-down method is applied based on historical proportions, so only the total series of export values needs to be modeled. This study implements the autoregressive integrated moving average (ARIMA) to model the total series of export values. The performance of the method is evaluated based on the out-of-sample mean absolute percentage error (MAPE). The results show that the MAPE for out-of-sample is 9.91% for top-down historical proportion based on ARIMA and 12.95% for ARIMA individual forecast. These results indicate that the top-down historical proportion based on ARIMA is highly accurate for forecasting the value of exports in Indonesia.

Keywords: Export, ARIMA, Hierarchical Time Series, Top-Down, Historical Proportion

Introduction

In the era of globalization, a country cannot stand alone without relations with other countries. This is as a result of the growing number and variety of people's needs that their own nation's production cannot satisfy. Therefore, it is necessary to have international trade. International trade activities can increase state income, foreign exchange reserves, and job opportunities. There are two types of activities in international trade transactions, namely exports and imports [1]. Export is the activity of removing goods from the Indonesian customs area to the customs area of another country. Export value is the net income obtained from all costs requested or which should be requested by the exporter, expressed in US dollars [2]. Exports as a source of foreign exchange are very much needed by countries with open economic systems because exports can make a major contribution to the stability and economic growth of the country [3]. Indonesia, as a country with abundant natural resources, plays an important role in the export sector. As recorded in the book The World Factbook published in 2022 [4], Indonesia is ranked 31st in the world as a country that carries out export activities with a total export of $136.8 billion. The development of exports in Indonesia in the period 2016–2021, taken from the publication of the Central Statistics Agency, is shown in the graph in Figure 1.

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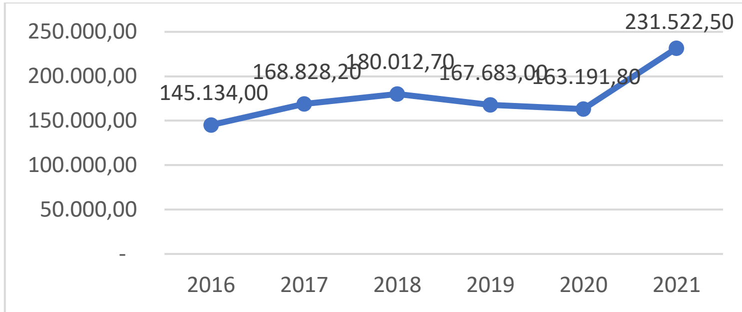

Figure 1. Series of export value in Indonesia (in million dollars)

Based on Figure 1, it can be seen that in the period 2016–2018, the export value increased annually to reach a value of US$ 180,012.7 million; however, in 2019 and 2020, the export value decreased to US$ 163,191.8 million. In 2021, export developments increased sharply, with an export value of US$ 231,522.5 million. As one of the factors supporting the country's foreign exchange, the decline and increase in the value of exports in Indonesia in the future need to be considered. The existence of forecasting the value of exports in Indonesia for some time in the future can certainly be used as a reference for the government in planning economic development in Indonesia. Therefore, an appropriate forecasting model is needed to determine whether the value of exports in Indonesia has increased or decreased.

Export value forecasting can be done using various time series methods. However, in this study, export value data are treated as hierarchical time series data. Data on the value of Indonesian exports has data levels. Total export value as top-level data or level 0 hierarchy, and export value for each province as bottom-level data or level 1 hierarchy In this research, the level 1 export value or export value for each province in Indonesia will be forecasted to find out what the forecasting results are for each province. For this reason, in this study, the hierarchical time series method was used. A hierarchical time series is a collection of several time series that are linked together in a hierarchical structure [5].

For hierarchical time series, forecasting methods are available, including bottom-up, top-down, and optimal combinations. To assess the effectiveness of the method, previous studies have been conducted [6–8]. The top-down method based on historical proportions is the most effective hierarchical forecasting method. This approach requires forecasting the entire aggregate series before breaking down the forecast by proportion. The top-down approach will be very helpful, especially if the aggregate series consists of elements with comparable patterns of variation [9]. For forecasting the total series, this study implemented Autoregressive Integrated Moving Average (ARIMA). ARIMA is a method that can forecast all types of data patterns, both seasonal and non-seasonal [10]. Apart from that, ARIMA can predict an event by looking at past data patterns quickly, simply, and accurately [10]. The performance of the method is evaluated based on the out-of-sample mean absolute percentage error (MAPE).

Materials and Methods

Materials

The data used in this study are secondary data obtained from the website of the Central Statistics Agency in Indonesia. The data are monthly export values in Indonesia. The data is divided into in-sample data for modeling and out-of-sample data for performance evaluation. The in-sample data is from January 2017 to December 2020, whereas the out-of-sample data is from January 2021 to December 2021. The data has two hierarchical levels. The "level 1" series is the export value for each province in Indonesia, and the "level 0" series is the aggregated data, which is the national export value. The hierarchical structure of the data is shown in Table 1.

Table 1. Hierarchical structure of data

Level	Series	Number of Series
0	Indonesia (Y_t)	1
1	Aceh ($Y_{1,t}$) Sumatera Utara ($Y_{2,t}$) Sumatera Barat ($Y_{3,t}$) ⋮ Maluku ($Y_{32,t}$) Papua ($Y_{33,t}$) Papua Barat ($Y_{34,t}$)	34

This study was conducted in several stages. The first is to determine hierarchical data from export values for both level 0 and level 1. Then, forecasting the total series (level 0) uses ARIMA. Forecasting the total series is then disaggregated into level 1 using the top-down method based on historical proportions. Forecasting results are used to evaluate the performance of the method based on the MAPE value. The stages are described in the flowchart in Figure 2.

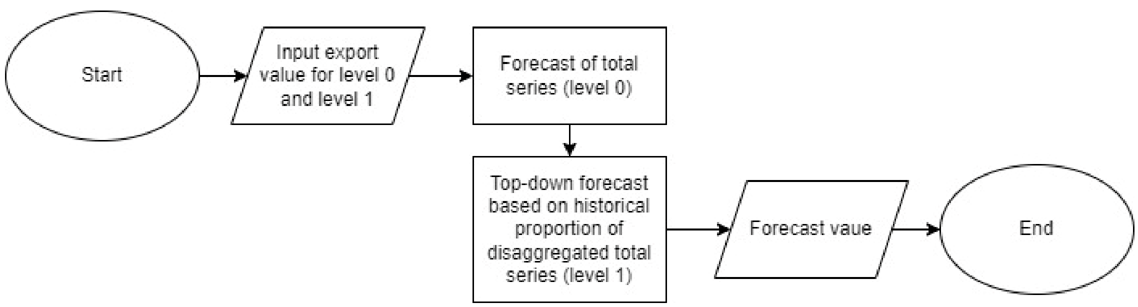

Figure 2. Flowchart of the study

Autoregressive Integrated Moving Average

The Autoregressive Integrated Moving Average (ARIMA) model makes predictions for the future based on a variety of observations from various prior time periods. This model can explain the results of autoregressive (AR) and moving average (MA). The most basic version of the ARIMA (p, d, q) model is

$$Y_t = \frac{\Theta(B)e_t}{\Phi(B)(1-B)^d} \quad \text{a)}$$

with $\Phi(B) = (1 - \varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_p B^p)$ and $\Theta(B) = (1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q)$, B is backshift operator, and e_t is a white noise process with zero mean and constant variance [11].

Hierarchical Time Series

Time series having a hierarchy and various levels are called hierarchical time series. According to Shang (2013), Figure 3 depicts the structure of a hierarchical time series [12].

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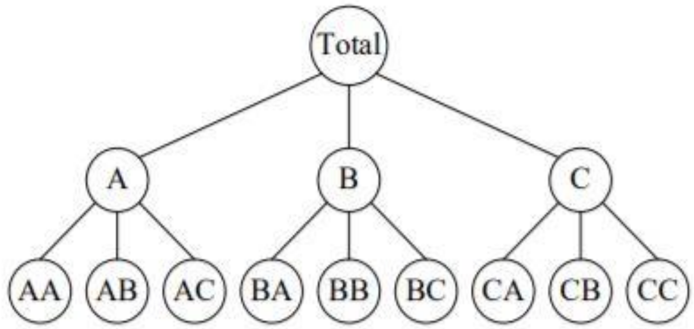

Figure 3. Hierarchical time series structure

The hierarchical time series is typically notated as $Y_{i,t}$, which includes all observations i and time t. As column vectors, all observations at timet marked by $Y_t = \left[Y_t, Y'_{1,t}, \cdots, Y'_{K,t}\right]^t$ are written as follows [6]:

$$Y_t = SY_{K,t} \tag{b)}$$

which can be elaborated to

$$\begin{bmatrix} Y_{Total,t} \\ Y_{A,t} \\ Y_{B,t} \\ Y_{C,t} \\ Y_{AA,t} \\ Y_{AB,t} \\ \vdots \\ Y_{CB,t} \\ Y_{CC,t} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ & & & & I_9 & & & & \end{bmatrix} \begin{bmatrix} Y_{AA,t} \\ Y_{AB,t} \\ Y_{AC,t} \\ Y_{BA,t} \\ Y_{BB,t} \\ Y_{BC,t} \\ Y_{CA,t} \\ Y_{CB,t} \\ Y_{CC,t} \end{bmatrix}$$

where S is the sum of the order $m\ x\ mK$ matrices which is the sum of all the lower level time series data [6]. The final forecast is obtained by using the following equation

$$\tilde{Y}_n(l) = SP\hat{Y}_n(l) \tag{c)}$$

where $\hat{Y}_n(l)$ is the basic prediction at all hierarchical levels, and P is the proportion matrix based on the hierarchical method used.

Top-Down Approach Based on Historical Proportion

According to a hierarchy of data proportions, the top-down historical proportion divides the estimated "total" series and distributes it downwardly [6]. Historical proportions come in two different varieties. The first has the following formula and is known as TDHP1 (top-down historical proportion 1)

$$p_i = \frac{1}{n}\sum_{t=1}^{n}\frac{Y_{i,t}}{Y_t}. \tag{d)}$$

While, the second is called TDHP2 (top-down historical proportion 2) with the following formula

$$p_i = \sum_{t=1}^{n}\frac{\frac{Y_{i,t}}{n}}{\frac{Y_t}{n}}. \tag{e)}$$

where $i = 1,2,\cdots,m$. The use of historical proportions has the advantage that the calculation is simple.

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Model Evaluation

Performance of time series models can be evaluated by using many kinds of measurement error. In this study, we use criteria of measurement error, that is mean absolute percentage error (MAPE). The formula of the MAPE is described as follows

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Y_t - \hat{Y}_t}{n}\right| \times 100\% \quad \text{f)}$$

where Y_t is the actual data, $\widehat{Y_t}$ is the predicted data, and n is the number observation.

Results and Discussions

This section describes the forecasting results of the total series and top-down forecasts based on the historical proportions of the total series that have been disaggregated.

ARIMA Modeling for Total Series

Indonesia's export value data is displayed in Figure 4. The time series plot shows that Indonesia's export value tends to increase every year, and there is a fluctuating pattern.

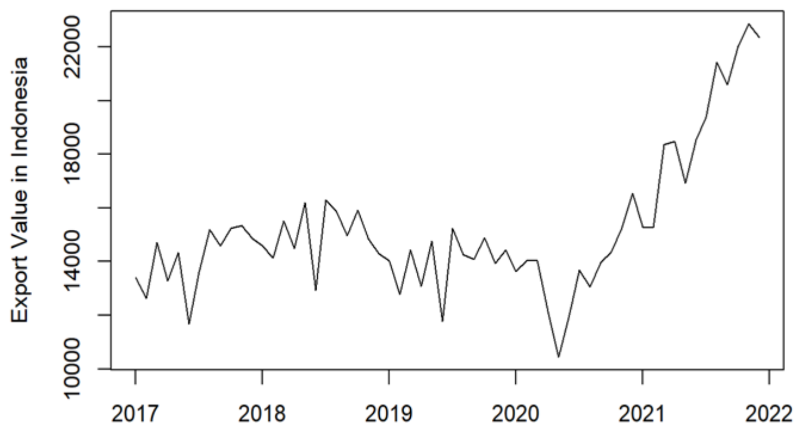

Figure 4. Time series plot of national export value in Indonesia

In this study, the study area covers Jakarta, Bogor, and Tangerang. This region is very interesting to study because it has a geographical area that is vulnerable to the influence of the MJO. This is supported by the topography of Jakarta, Bogor and Tangerang, and Bekasi which are hilly and surrounded by the sea. This varied topography has resulted in a variety of rainfall distribution patterns in the areas of Jakarta, Bogor, Depok, Tangerang, and Bekasi [13].

By using ARIMA, the best model for total of Indonesia's export value (level 0) is ARIMA(1,2,1), with the following description

$$Y_t = \frac{(1 - B)}{(1 + 0.56B)(1 - B)^2} e_t.$$

Top-Down Based on Historical Proportion for Each Province

In this stage, the total series is disaggregated into level 1, which is, export value for each province using top-down approach based on historical proportion. The historical proportion is obtained using Equation (4). The results of the proportions obtained for each province in Indonesia are shown in Table 2.

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Table 2. Historical proportion of top-down approach

Series	Proportion of Top-Down Approach	Series	Proportion of Top-Down Approach
Aceh	0.00142	Nusa Tenggara Timur	0.00010
Sumatera Utara	0.05007	Nusa Tenggara Barat	0.00383
Sumatera Barat	0.01027	Kalimantan Barat	0.00650
Riau	0.08626	Kalimantan Tengah	0.01177
Kep. Riau	0.07294	Kalimantan Selatan	0.04127
Jambi	0.01402	Kalimantan Timur	0.09672
Sumatera Selatan	0.02391	Kalimantan Utara	0.00612
Bengkulu	0.00127	Sulawesi Utara	0.00500
Lampung	0.02007	Sulawesi Tengah	0.03579
Kep. Bangka Belitung	0.00943	Sulawesi Selatan	0.00668
DKI Jakarta	0.30767	Sulawesi Tenggara	0.01038
Jawa Barat	0.16582	Sulawesi Barat	0.00316
Jawa Tengah	0.04369	Gorontalo	0.00209
DI Yogyakarta	0.00238	Maluku Utara	0.00671
Jawa Timur	0.11335	Maluku	0.00031
Banten	0.06463	Papua	0.01449
Bali	0.00300	Papua Barat	0.01231

Based on the historical proportions summarized in Table 2, it is possible to forecast the export value of each province (level 1). The model obtained is presented in Table 3.

Table 3. Top-down historical proportion based on ARIMA model for each province

Series	Model	Description
Aceh	ARIMA(0,2,1)	$Y_{1,t} = \frac{(1-B)}{(1-B)^2} e_{1,t}$
Sumatera Utara	ARIMA(0,2,1)	$Y_{2,t} = \frac{(1-0.9473B)}{(1-B)^2} e_{2,t}$
Sumatera Barat	ARIMA(0,2,1)	$Y_{3,t} = \frac{(1-0.9789B)}{(1-B)^2} e_{3,t}$
Riau	ARIMA(2,2,0)	$Y_{4,t} = \frac{1}{(1+0.4013B+0.4108B^2)(1-B)^2} e_{4,t}$
Kep. Riau	ARIMA(2,2,0)	$Y_{5,t} = \frac{1}{(1+0.4011B+0.3916B^2)(1-B)^2} e_{5,t}$
Jambi	ARIMA(2,2,0)	$Y_{6,t} = \frac{1}{(1+0.4013B+0.4110B^2)(1-B)^2} e_{6,t}$
Sumatera Selatan	ARIMA(2,2,0)	$Y_{7,t} = \frac{1}{(1+0.3486B+0.4107B^2)(1-B)^2} e_{7,t}$
Bengkulu	ARIMA(2,2,0)	$Y_{8,t} = \frac{1}{(1+0.4023B+0.4105B^2)(1-B)^2} e_{8,t}$
Lampung	ARIMA(0,2,1)	$Y_{9,t} = \frac{(1-B)}{(1-B)^2} e_{9,t}$
Kep. Bangka Belitung	ARIMA(0,2,1)	$Y_{10,t} = \frac{(1-B)}{(1-B)^2} e_{10,t}$
DKI Jakarta	ARIMA(2,2,0)	$Y_{11,t} = \frac{1}{(1+0.4023B+0.4108B^2)(1-B)^2} e_{11,t}$
Jawa Barat	ARIMA(0,2,1)	$Y_{12,t} = \frac{(1-0.9577B)}{(1-B)^2} e_{12,t}$
Jawa Tengah	ARIMA(0,2,1)	$Y_{13,t} = \frac{(1-0.9051B)}{(1-B)^2} e_{13,t}$
DI Yogyakarta	ARIMA(0,2,1)	$Y_{14,t} = \frac{(1-B)}{(1-B)^2} e_{14,t}$

Series	Model	Description
Jawa Timur	ARIMA(0,2,1)	$Y_{15,t} = \frac{(1-0.9250B)}{(1-B)^2} e_{15,t}$
Banten	ARIMA(2,2,0)	$Y_{16,t} = \frac{1}{(1+0.4023B+0.4108B^2)(1-B)^2} e_{16,t}$
Bali	ARIMA(0,2,1)	$Y_{17,t} = \frac{(1-B)}{(1-B)^2} e_{17,t}$
Nusa Tenggara Timur	ARIMA(2,2,0)	$Y_{18,t} = \frac{1}{(1+0.3813B+0.4237B^2)(1-B)^2} e_{18,t}$
Nusa Tenggara Barat	ARIMA(2,2,0)	$Y_{19,t} = \frac{(1-0.7125B)}{(1-B)^2} e_{19,t}$
Kalimantan Barat	ARIMA(0,2,1)	$Y_{20,t} = \frac{1}{(1+0.4021B+0.4107B^2)(1-B)^2} e_{20,t}$
Kalimantan Tengah	ARIMA(0,2,1)	$Y_{21,t} = \frac{(1-B)}{(1-B)^2} e_{21,t}$
Kalimantan Selatan	ARIMA(0,2,1)	$Y_{22,t} = \frac{(1-0.8891B)}{(1-B)^2} e_{22,t}$
Kalimantan Timur	ARIMA(2,2,0)	$Y_{23,t} = \frac{1}{(1+0.4013B+0.4108B^2)(1-B)^2} e_{23,t}$
Kalimantan Utara	ARIMA(2,2,0)	$Y_{24,t} = \frac{1}{(1+0.4009B+0.4110B^2)(1-B)^2} e_{24,t}$
Sulawesi Utara	ARIMA(0,2,1)	$Y_{25,t} = \frac{(1-B)}{(1-B)^2} e_{25,t}$
Sulawesi Tengah	ARIMA(0,2,1)	$Y_{26,t} = \frac{(1-0.9171B)}{(1-B)^2} e_{26,t}$
Sulawesi Selatan	ARIMA(2,2,0)	$Y_{27,t} = \frac{1}{(1+0.4012B+0.4110B^2)(1-B)^2} e_{27,t}$
Sulawesi Tenggara	ARIMA(1,2,0)	$Y_{28,t} = \frac{1}{(1+0.5108B)(1-B)^2} - e_{28,t}$
Sulawesi Barat	ARIMA(0,2,1)	$Y_{29,t} = \frac{(1-B)}{(1-B)^2} e_{29,t}$
Gorontalo	ARIMA(0,2,1)	$Y_{30,t} = \frac{(1+B)}{(1-B)^2} e_{30,t}$
Maluku Utara	ARIMA(0,2,1)	$Y_{31,t} = \frac{(1-0.8836B)}{(1-B)^2} e_{31,t}$
Maluku	ARIMA(0,2,1)	$Y_{32,t} = \frac{(1-0.9901B)}{(1-B)^2} e_{32,t}$
Papua	ARIMA(2,2,0)	$Y_{33,t} = \frac{1}{(1+0.4023B+0.4107B^2)(1-B)^2} e_{33,t}$
Papua Barat	ARIMA(0,2,1)	$Y_{34,t} = \frac{(1-0.3976B)}{(1-B)^2} e_{34,t}$

Based on the model obtained, as summarized in Table 3, the export value of each province can be predicted. The results of this forecasting are then used to determine the performance of the method used. MAPE from in-sample data and out-of-sample data is presented in Table 4.

Table 4. MAPE Value of Top-Down and Individual Forecast

Method	In-sample (%)	Out-of-Sample(%)
ARIMA individual forecast	11.85	12.95
Top-down historical proportion based on ARIMA	3.94	9.91

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Table 4 shows the MAPE of the out-of-sample data of less than 20% for the ARIMA individual forecast. Following [13], it shows that the performance of the method for forecasting is good. Even for top-down historical proportion based on ARIMA, the performance of the method is highly accurate because it has a MAPE value below 10% [13].

Based on the best model, forecasting of the export value of each province in Indonesia can be made. For example, the results of forecasting export values for Aceh Province using the top-down historical proportion based on ARIMA are shown in Figure 5.

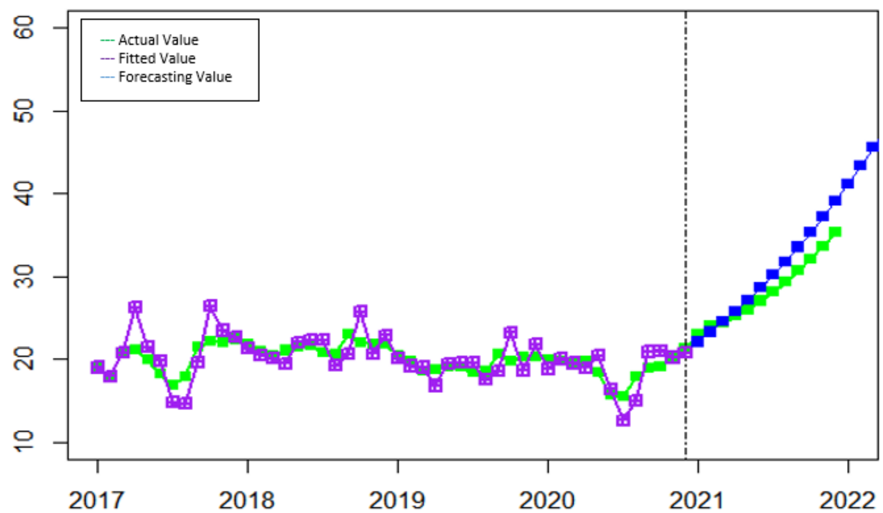

Figure 5. Forecasting of export value in Aceh Province

Based on the results of forecasting export values for Aceh province in Figure 5, it is found that the forecasting results for the future period for Aceh province have consistently increased.

Conclusion

In this study, export data has a hierarchical structure. Based on the results, the top-down approach based on historical proportions is good for predicting export values, which are data with a hierarchical structure. This is indicated by the small MAPE value for both in-sample data of 3.94% and out-of-sample data of 9.91%. By using the best method, forecasting of the export value of each province in Indonesia can be made. For example, in Aceh Province, the forecasting results for the future period have consistently increased

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