ON THE CALCULATION OF IMPLIED VOLATILITY USING A GENETIC ALGORITHM

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ABSTRACT

In the Black-Scholes options pricing formulas one parameter that cannot be directly observed is the volatility of the stock price. If actual market data of the price V are known, then the volatility can be viewed as unknown and can be calculated via the implicit equation $V - v(S,T,t,K,r,\sigma) = 0$.

The volatility σ *plays the role of the unknown parameter. The volatility* σ *determined in this way is called implied volatility and is the root of the equation* $f(\sigma) = V - v(S,T,t,K,r,\sigma) = 0$. *Iterative methods such as Newton's method, can then be used to find the root. In this work we propose an approach that uses a genetic algorithm to find the implied volatility.*

Keywords: *option pricing, implied volatility, genetic algorithm*

1. Introduction

The two types of option contracts that are most common are *calls* and *puts.* A call is an option to buy, and a put is an option to sell. A *call option* (*put option*) gives the holder the right, but not the obligation, to buy from (sell to) the writer a stock at an agreed price (known as the *exercise price* or the *strike price*), with that right lasting until a particular date (known as the *maturity* or the *expiration date*). The options that can be exercised at any time before or on the expiration date are called *American options*. The options that can only be exercised at their expiration date are called *European options*. In this work we are particularly concern with the European call option.

If we let $S(t)$ denote the stock price at time t, K denote the strike price and T the expiration time, then the *intrinsic value* of the European call option is $\max(S(T)-K,0)$. This is because: if $S(T) > K$ the option will be exercised for a profit of $S(T)-K$, and if $S(T) \le K$ the option will not be exercised. Because the holder of the call option is not obliged to buy the stock, he does not lose any money (in the first case he gained money and in the second case he neither gain nor lose). In contrast, the writer of the call option will not gain any money at the expiration date, and may lose an unlimited amount. To compensate for this imbalance, when the option is agreed (today) the holder of the option would be expected to pay to the writer of the option an amount of money as the *value* of the option.

Options have become so popular that in many cases more money is invested in them. At least there are two reasons for that (Higham, 2004):

a. Options are extremely attractive to investors, both for *speculation* and for *hedging*.
b. There is a systematic way to determine how much they worth, and hence they can be bought and sold with some confidence.

Black and Scholes (1973) were the first to provide a closed-form solution for the valuation of European options. Their theory models the stock price as a stochastic process. Using a number of simplifying assumptions about the option market and the *no-arbitrage principle* in economics, they come up with the following formula for the value of a European call option:

$$C(S,t) = S\,N(d_1) - K\,e^{-r(T-t)}N(d_2) \qquad (0.1)$$

where

$$d_1 = \frac{\ln(S/K) + \left(r + \tfrac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

and $N(.)$ is the cumulative distribution function for the standardized normal distribution,

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{\varsigma^2}{2}}\,d\varsigma\,.$$

Equation (1.1) says that the price of a call option on a stock which pays no dividends depends on only five variables: the stock's price (*S*), the strike price (*K*), the time to maturity (*T* – *t*), the continuously compounded interest rate (*r*) and the volatility of the stock (σ).

There is a relationship, called *put-call parity*, between the value *C* of a European call option and the value *P* of a European put option with the same strike price *K* and expiration date *T*. At time $t = 0$, we have

$$C + K\,e^{-rT} = P + S \qquad (0.2)$$

Hence the put value may be obtained by using equation (1.2) and equation (1.1).

The one parameter in the Black-Scholes pricing formula (1.1) that can not be directly observed is the volatility of the stock. The volatility of a stock is a measure of the uncertainty of the return provided by the stock. It can be estimated from historical data of the stock price or, in practice, implied from option prices observed in the market. In this work we will be concern with the latter. For the use of historical data to estimate the volatility see for example Hull (2003), Higham (2004) and Ross (2003).

2. Implied Volatility

We have seen that the Black-Scholes call and put values depend on $S, K, r, T-t$ and σ; only the stock volatility σ that can not be observed directly. One approach to obtain the volatility is to extract it from the observed marked data, that is: given a quoted option value, and knowing S, K, r, t and T, find the σ that leads to this value. Having found σ we may use the Black-Scholes formula to value other options on the same stock. The volatility σ computed in this way is known as an *implied volatility*. In this work we focus our attention of extracting σ from a European call option quote.

Assuming that the parameters S, K, r, T and t are known, we can consider the option value as a function of σ only and denote it by $C(\sigma)$. If we are given a quoted call value C^*, then the problem is to find the implied volatility σ^* that solves $C(\sigma) = C^*$. Since σ can not be solved explicitly in terms of S, K, r, T, t and the value of the call option C from the formula (1.1), the determination of σ must be performed as a root-finding problem for a non-linear equation.

Let's write our non-linear equation for σ^* as

$$f(\sigma) = 0 \text{ where } f(\sigma) = C(\sigma) - C^* \qquad (2.1)$$

Several iterative root-finding techniques for non-linear equation are available, such as Bisection, Secant and Newton's methods. All the methods begin with an initial approximation for the root and generate a sequence that converges to a root of the equation, if the method is successful. Rapid convergence is generally obtained using the Newton's method which requires a good initial approximation of the root. A good source and analysis of the use of Newton's method for the calculation of implied volatility are Kwok (1999) and Higham (2004). In this work we propose the use of a genetic algorithm as a simple method for the calculation of implied volatility. Chen and Lee (1997) illustrated and tested the ability of Genetic Algorithm to determine the price of European Call options

3. A Genetic Algorithm approach

Genetic Algorithms are a class of algorithms inspired by evolution. These algorithms encode solution to a specific problem on a chromosome like data structure and apply recombination operators to these structures so as to preserve critical information. An implement-ation of a Genetic Algorithm begins with a population of random chromosome. The members of this popula-tion are usually strings of zeros and ones. Each string in the population corresponds to a chromosome and each binary element of the string to a gene. Instead of binary coding, one can also consider a real coding. This initial population is generated randomly. The chromosomes are then evaluated and allocated repro-ductive opportunities in such a way that those which represent a better solution to the problem are given more chances to reproduce. Following are the basic steps in a Genetic Algorithm:

a. Generate randomly an initial population of chromosomes.
b. Calculate the fitness, defined according to some specified criteria, of all the members of the population and select individuals for the reproduction process. The fittest are given a greater probability of reproducing in propor-tion to the value of their fitness.
c. Apply the genetic operators of crossover and mutation to the selected individuals to create new individuals and thus a new generation. Crossover exchanges some of the bits (genes) of the two chromosomes, whereas mutation inverts any bit(s) of the chromosome depending on a probability of mutation. Thus a 0 may be changed to a 1 or vice versa.

Then again step 2 is followed until the condition for ending the algorithm is reached.

Traditionally Genetic Algorithms have been used for optimization problems. Among the advantages of applying Genetic Algorithms to optimization problems is that Genetic Algorithms do not have much mathe-matical requirements about the optimization problems. Due to their evolutionary nature, Genetic Algorithms will search for solution without regard to the specific inner workings of the problem. Genetic Algorithms can handle any kind of objective functions and any kind of constraints (i.e. linear or nonlinear) defined on discrete or continuous search spaces (Gen and Cheng, 1997). Genetic Algorithms can also be used to the root-finding problem of transcendental equation as de-monstrated by Aggarwal (2003), by first converting the problem into an optimization problem. The tech-nique can be generalized to the system of non-linear equations. That is we can convert the problem of find-ing the roots of system of nonlinear equations into a multi-modal optimization problem and then used a Ge-netic Algorithm to solve the optimization problem and hence obtain the roots of the corresponding system of nonlinear equations (Sidarto et.al., 2004).

In this work we use a simple Genetic Algorithm to find an approximate solution of the nonlinear equation expressed by equation (2.1). Binary encoding was used to this problem and the fitness function chosen was

$$F(\sigma) = \frac{1}{1 + |f(\sigma)|} \qquad (3.1)$$

Hence at the vicinity of the root $f(\sigma) = 0$ we expect that the value of the fitness function will be close to 1 and will be a small positive number in the case σ is far from the root. Thus our problem of finding a solution to equation (1.3) initially will be formulated as a problem of finding σ that maximizes the function $F(\sigma)$ in equation (3.1).

4. Example

As a first example we consider the following data taken from Ross (2003):

$S = 30, K = 34, r = 0.08, s = 0.2, t = 0$ and $T = 0.25$

in order to compute the Black-Scholes value for C by equation (1.1), and then using the Genetic Algorithm to found the value s. Using equation (1.1) we found $C^* = 0.2383$. Hence if we consider $C^* = 0.2383$ as 'market' data for a call option for which

$S = 30,\ K = 34, r = 0.08, t = 0$ and $T = 0.25$

our problem is to find s^* as a root of equation (2.1).
In this problem we have used as Genetic Algorithm parameters: population size = 100, chromosome length = 16, probability of (one point) crossover = 0.8, probability of mutation = 0.005., maximum number of generation = 100. As a result we obtained the implied volatility $s = 0.1998$.

For the second example the data was taken from Higham (2004). This is the data for call options traded on the LIFFE as reported in the Financial Times on August 22, 2001. The data is for the FTSE 100 index, which is an average of 100 stocks quoted on the London Stock Exchange. The expiry date for these options was December 2001. On August 22, 2001 the stock price was 5420.3. The values for r and T were $r = 0.05$ and $T = 4/12$ for the duration of option. The first column of table 1 shows the eight different strike prices and the second column its corresponding call option prices. The last column shows the implied volatility computed for each of the eight different strike prices using Genetic Algorithm with the same parameters as used for the first example.

Table 1.

Strike price	***Option price***	***Implied Volatility***
5125	475	0.1981
5225	405	0.1960
5325	340	0.1934
5425	280.5	0.1903
5525	226	0.1862
5625	179.5	0.1833
5725	139	0.1799
5825	105	0.1766

Figure 1 shows the implied volatility computed for the eight different strike prices as in Table 1. We see that the curve is convex in shape, rather than straight horizontal line as suggested by the Black-Scholes formula. If the Black-Scholes formula were valid, the volatility would be the same for each strike price.

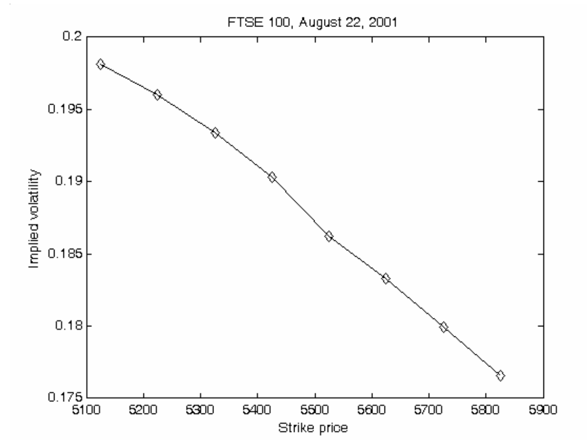

Figure 1

5. Conclusion

An approach to calculate implied volatility using a simple Genetic Algorithm was presented. The method is easy to implement without using much mathematical requirements.

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